Thermal Design

Heat Transfer
Temperature Measurement
The prevalence of the number 5.7

Why Care about Thermal?

- Scientific equipment often needs temperature control
  - especially in precision measurement
- Want to calculate thermal energy requirements
  - how much energy to change temperature?
  - how much power to maintain temperature?
- Want to calculate thermal time constants
  - how long will it take to change the temperature?
- Want to understand relative importance of radiation, convection, conduction
  - which dominates?
  - how much can we limit/exaggerate a particular process?

Chief Thermal Properties

- Thermal Conductivity
  - κ measured in W/m/K
  - heat flow (in W) is

    $$P = \kappa \cdot \Delta T \cdot A/t$$
  - note that heat flow increases with increasing ΔT, increasing surface area, and decreasing thickness (very intuitive)
- Specific Heat Capacity
  - c_p measured in J/kg/K
  - energy locked up in heat is:

    $$E = c_p \cdot \Delta T \cdot m$$
  - energy stored proportional to ΔT, and mass (intuitive)
- Emisivity, ε
  - power radiated is $P = \varepsilon A \sigma T^4$

Thermal Conductivity of Materials

- (copied from materials lecture)

Material	κ (W m^{-1} K^{-1})	comments
Silver	422	room T metals feel cold
Copper	391	great for pulling away heat
Gold	295	
Aluminum	205	
Stainless Steel	10–25	why cookware uses S.S.
Glass, Concrete,Wood	0.5–3	buildings
Many Plastics	~0.4	room T plastics feel warm
G-10 fiberglass	0.29	strongest insulator choice
Stagnant Air	0.024	but usually moving…
Styrofoam	0.01–0.03	can be better than air!

Conduction: Heated Box

- A 1 m × 1 m × 2.5 m ice-fishing hut stands in the -10° C cold with 2.5 cm walls of wood
  - A = 12 m^2
  - t = 0.025 m
  - $\kappa \approx 1$ W/m/K
- To keep this hut at 20° C would require

  $$P = \kappa \cdot \Delta T \cdot A/t = (1.0)(30)(12)/(0.025) = 14{,}400 \text{ W}$$

  - Outrageous!
  - Replace wood with insulation: $\kappa = 0.02$; $t = 0.025$

    $$P = \kappa \cdot \Delta T \cdot A/t = (0.02)(30)(12)/(0.025) = 288 \text{ W}$$

  - This, we can do for less than $40 at Target
- First example unfair
  - air won't carry heat away this fast: more on this later

A Cold Finger

- Imagine a plug of aluminum connecting the inside to the outside
  - how much will it change the story?
  - cylindrical shape, length t, radius R
  - κ = 205 W/m/K
  - just based on conduction alone, since difference in thermal conductivity is a factor of 10,000, the cold finger is as important as the whole box if it's area is as big as 1/10,000 the area of the box.
    - this corresponds to a radius of 2 mm !!!
- So a cold finger can "short-circuit" the deliberate attempts at insulation
  - provided that heat can couple to it effectively enough: this will often limit the damage

R-value of insulation

- In a hardware store, you'll find insulation tagged with an "R-value"
  - thermal resistance R-value is t/κ
  - R-value is usually seen in imperial units: $\text{ft}^2\cdot\text{F}\cdot\text{hr/Btu}$
  - Conversion factor is 5.67:
    - R-value of 0.025-thick insulation of $\kappa = 0.02$ W/m/K is:

      $R = 5.67 \times t/\kappa = 5.67 \times 0.025/0.02 = 7.1$
  - Can insert Home-Depot R=5 insulation into formula:

    $$P = 5.67 \times A\cdot\Delta T/R$$
  - so for our hut with $R = 5$: $P \approx 5.67 \times (12)(30)/5 = 408$ W
  - note our earlier insulation example had $R = 7.1$ instead of 5, in which case $P = 288$ W (check for yourself!)

Wikipedia on R-values:

- Note that these examples use the non-SI definition and are per inch. Vacuum insulated panel has the highest R-value of (approximately 45 in English units) for flat, Aerogel has the next highest R-value 10, followed by isocyanurate and phenolic foam insulations with, 8.3 and 7, respectively. They are followed closely by polyurethane and polystyrene insulation at roughly R–6 and R–5. Loose cellulose, fiberglass both blown and in batts, and rock wool both blown and in batts all possess an R-value of roughly 3. Straw bales perform at about R–1.45. Snow is roughly R–1.
- Absolutely still air has an R-value of about 5 but this has little practical use: Spaces of one centimeter or greater will allow air to circulate, convecting heat and greatly reducing the insulating value to roughly R–1

Convective Heat Exchange

- Air (or any fluid) can pull away heat by physically transporting it
  - really conduction into fluid accompanied by motion of fluid
  - full, rigorous, treatment beyond scope of this class
- General behavior:

  power convected = $P = h \cdot \Delta T \cdot A$

  - A is area, ΔT is temperature difference between surface and bath
  - h is the convection coefficient, units: $W/K/m^2$
  - still air has $h \approx 2–5$ $W/K/m^2$
    - higher when ΔT is higher: self-driven convective cells
    - note that $h = 5.67$ is equivalent to $R = 1$
  - gentle breeze may have $h \approx 5–10$ $W/K/m^2$
  - forced air may be several times larger ($h \approx 10–50$)

Convection Examples

- Standing unclothed in a 20° C light breeze
  - $h \approx 5$ W/K/m^2
  - $\Delta T = 17°$ C
  - $A \approx 1$ m^2
  - $P \approx (5)(17)(1) = 85$ W
- Our hut from before:
  - $h \approx 5$ W/K/m^2
  - $\Delta T = 30°$ C (if the skin is at the hot temperature)
  - $A \approx 12$ m^2
  - $P \approx (5)(30)(12) = 1800$ W

Radiative Heat Exchange

- The Stephan-Boltzmann law tells us:
  - $P = \varepsilon A\sigma(T_h^4 - T_c^4)$
  - The Stephan-Boltzmann constant, $\sigma = 5.67\times10^{-8}$ W/m^2/K^4
  - in thermal equilibrium ($T_h = T_c$), there is radiative balance, and $P = 0$
  - the emissivity ranges from 0 (shiny) to 1 (black)
  - "black" in the thermal infrared band ($\lambda \approx 10\ \mu$m) might not be intuitive
    - your skin is nearly black ($\varepsilon \approx 0.8$)
    - plastics/organic stuff is nearly black ($\varepsilon \approx 0.8$–1.0)
    - even white paint is black in the thermal infrared
    - metals are almost the only exception
  - for small ΔT around T, $P \approx 4\varepsilon A\sigma T^3\,\Delta T = (4\varepsilon\sigma T^3)\cdot A\cdot\Delta T$
  - which looks like convection, with $h = 4\varepsilon\sigma T^3$
  - for room temperature, $h \approx 5.7\varepsilon$ W/K/m^2, so similar in magnitude to convection

Radiative Examples

- Standing unclothed in room with -273° C walls
  - and assume emissivity is 0.8 for skin
  - $A \approx 1\ \text{m}^2$
  - ΔT = 310 K
  - $P \approx (0.8)(1)(5.67\times10^{-8})(310^4) = 419$ W (burr)
- Now bring walls to 20° C
  - ΔT = 17° C
  - $P \approx (0.8)(1)(5.67\times10^{-8})(310^4 - 293^4) = 84$ W
  - pretty similar to convection example
  - note that we brought our cold surface to 94.5% the absolute temperature of the warm surface, and only reduced the radiation by a factor of 5 (not a factor of 18): the fourth power makes this highly nonlinear

Combined Problems

- Two-layer insulation
  - must compute temperature at interface
- Conduction plus Convection
  - skin temperature must be solved
- Conduction plus Radiation
  - skin temperature must be solved
- The whole enchilada
  - conduction, convection, radiation

Two-Layer insulation

- Let's take our ice-fishing hut and *add* insulation instead of replacing the wood with insulation
  - each still has thickness 0.025 m; and surface area 12 m^2
  - Now have three temperatures: $T_{in} = 20°$, T_{mid}, $T_{out} = -10°$
  - Flow through first is: $P_1 = \kappa_1 \cdot (T_{in} - T_{mid}) \cdot A_1/t_1$
  - Flow through second is: $P_2 = \kappa_2 \cdot (T_{mid} - T_{out}) \cdot A_2/t_2$
  - In thermal equilibrium, must have $P_1 = P_2$
    - else energy is building up or coming from nowhere
  - We know everything but T_{mid}, which we easily solve for:

    $T_{mid}(\kappa_1 A_1/t_1 + \kappa_2 A_2/t_2) = \kappa_1 A_1 T_{in}/t_1 + \kappa_2 A_2 T_{out}/t_2$
  - find $T_{mid} = -9.412$ or $T_{mid} = 19.412$ depending on which is interior or exterior
  - heat flow is 282 W (compare to 288 W before: wood hardly matters)

Conduction plus Convection

- Let's take our hut with just wood, but considering convection
  - The skin won't necessarily be at T_{out}
  - Again, thermal equilibrium demands that power conducted through wall equals power wafted away in air
  - $P = h\cdot(T_{skin} - T_{out})\cdot A = \kappa\cdot(T_{in} - T_{skin})\cdot A/t$
  - for which we find $T_{skin} = (\kappa T_{in}/t + hT_{out})/(h + \kappa/t) = 16.7°$ C
  - so the skin is hot
  - $P = (5)(26.7)(12) \approx 1600$ W
  - So a space heater actually could handle this (no radiation)
  - lesson: air could not carry heat away fast enough, so skin warms up until it can carry enough heat away—at the same time reducing ΔT across wood
  - h may tend higher due to self-induced airflow with large ΔT
  - also, a breeze/wind would help cool it off

Convection plus Radiation

- How warm should a room be to stand comfortably with no clothes?
  - assume you can put out P = 100 W metabolic power
    - 2000 kcal/day = 8,368,000 J in 86400 sec ≈ 100 W
  - $P = h\cdot(T_{skin} - T_{out})\cdot A + \varepsilon A\sigma(T_{skin}^4 - T_{out}^4) \approx (hA + 4\varepsilon A\sigma T^3)\Delta T$
  - with emissivity = 0.8, T = 293 K
  - $100 = ((5)(1) + 4.56)\Delta T$
  - $\Delta T = 10.5°$
  - so the room is about 310 - 10.5 = 299.5 K = 26.5° C = 80° F
  - iterating (using T = 299.5); 4.56 → 4.87; ΔT → 10.1°
  - assumes skin is full internal body temperature
    - some conduction in skin reduces skin temperature
    - so could tolerate slightly cooler

The whole enchilada

- Let's take a cubic box with a heat source inside and consider all heat transfers
  - P = 1 W internal source
  - inside length = 10 cm
  - thickness = 2.5 cm
  - R-value = 5
  - so 5.67×t/κ = 5 → κ = 0.028 W/m/K
  - effective conductive area is 12.5 cm cube → A_c = 0.09375 m^2
  - external (radiative, convective) area is 15 cm cube → A_{ext} = 0.135 m^2
  - assume h = 5 W/K/m^2, ε = 0.8, T_{ext} = 293 K
  - assume the air inside is thoroughly mixed (perhaps 1 W source is a fan!)

The enchilada calculation

- power generated = power conducted = power convected plus power radiated away

  $P = \kappa \cdot (T_{in} - T_{skin}) \cdot A_c / t = (hA_{ext} + 4\varepsilon A_{ext}\sigma T^3) \cdot (T_{skin} - T_{ext})$

  – first get T_{skin} from convective/radiative piece
  – $T_{\text{skin}} = T_{\text{ext}} + P / (hA_{ext} + 4\varepsilon A_{ext}\sigma T^3) = 20° + 1.0/(0.675+0.617)$
  – $T_{\text{skin}} = 20.8°$ (barely above ambient)
  – now the ΔT across the insulation is $P \cdot t / (A_c \cdot \kappa) = 9.5°$
  – so $T_{\text{in}} = 30.3°$
- Notice a few things:
  – radiation and convection nearly equal influence (0.617 vs. 0.675)
  – shutting off either would result in small (but measurable) change

Timescales

- So far we've looked at steady-state equilibrium situations
- How long will it take to "charge-up" the system?
- Timescale given by heat capacity times temperature change divided by power
  - $\tau \approx c_p \cdot m \cdot \Delta T / P$
- For ballpark, can use $c_p \approx 1000$ J/kg/K for just about anything
  - so the box from before would be 2.34 kg if it had the density of water; let's say 0.5 kg in truth
  - average charge is half the total ΔT, so about 5°
  - total energy is (1000)(0.5)(5) = 2500 J
  - at 1W, this has a 40 minute timescale

Heating a lump by conduction

- Heating food from the outside, one relies entirely on thermal conduction/diffusion to carry heat in
- Relevant parameters are:
  - thermal conductivity, κ (how fast does heat move) (W/m/K)
  - heat capacity, c_p (how much heat does it hold) (J/kg/K)
  - mass, m (how much stuff is there) (kg)
  - size, R—like a radius (how far does heat have to travel) (m)
- Just working off units, derive a timescale:
  - $\tau \approx (c_p/\kappa)(m/R) \approx 4(c_p/\kappa)\rho R^2$
  - where ρ is density, in kg/m^3: $\rho \approx m/((4/3)\pi R^3) \approx m/4R^3$
  - faster if: c_p is small, κ is large, R is small (these make sense)
  - for typical food values, $\tau \approx 6$ minutes $\times$ $(R/1\text{ cm})^2$
  - egg takes ten minutes, turkey takes 5 hours

Lab Experiment

- We'll build boxes with a heat load inside to test the ideas here
- In principle, we can:
  - measure the thermal conductivity of the insulation
  - see the impact of emissivity changes
  - see the impact of enhanced convection
  - look for thermal gradients in the absence of circulation
  - look at the impact of geometry on thermal state
  - see how serious heat leaks can be
- Nominal box:
  - 10 cm side, 1-inch thick, about 1.5 W (with fan)

Lab Experiment, cont.

- We'll use power resistors rated at 5 W to generate the heat
  - 25 Ω nominal
  - $P = V^2/R$
  - At 5 V, nominal value is 1 W
  - can go up to 11 V with these resistors to get 5 W
  - a 12 Ω version puts us a bit over 2 W at 5 V
- Fans to circulate
  - small fans operating at 5 V (and about 0.5 W) will keep the air moving
- Aluminum foil tape for radiation control
  - several varieties available
- Standard building insulation

Lab Experimental Suite

experiment	R	int. airflow	ext. airflow	int. foil	ext. foil	geom.
A (control)	25 Ω	1 fan	none	no	no	10 cm cube
B (ext. convec)	25 Ω	1 fan	fan	no	no	10 cm cube
C (ext. radiation)	25 Ω	1 fan	none	no	yes	10 cm cube
D (ext. conv/rad)	25 Ω	1 fan	fan	no	yes	10 cm cube
E (gradients)	25 Ω	none	none	no	no	10 cm cube
F (int. radiation)	25 Ω	1 fan	none	yes	no	10 cm cube
G (radiation)	25 Ω	1 fan	none	yes	yes	10 cm cube
H (more power)	12 Ω	1 fan	none	no	no	10 cm cube
I (larger area)	12 Ω	2 fans	none	no	no	17.5 cm cube
J (area and thick.)	12 Ω	2 fans	none	no	no	17.5 cm cube

Random Notes

- Rig fan and resistor in parallel, running off 5V
  - fan can accept range: 4.5–5.5 V
  - if you want independent control, *don't* rig together
- Use power supply current reading (plus voltage) to ascertain power ($P = IV$) being delivered into box
- Make sure all RTDs read same thing on block of thermally stabilized chunk of metal
  - account for any offset in analysis
- Don't let foil extend to outside as a cold finger
- Make sure you have no air gaps: tape inside and out of seams
  - but need to leave top accessible
  - nice to tape fan to top (avoid heat buildup here)
  - can hang resistor, RTD from top as well (easy to assemble)

Random Notes, continued

- Measure temp. every 15 seconds, initially
  - tie white leads of RTDs to common DVM all together
  - label red lead so you know where it goes
- After equilibrium is reached, measure skin temperatures
  - hold in place with spare foam (not finger or thermal conductor!)
- We have limited RTDs, so 3−4 per group will be standard
  - locate inside RTD in fan exhaust, so representative
  - use external RTD for ambient, skin (double duty)
  - some experiments will want more RTDs (gradients)
- Once equilibrated, go to configuration B
  - turn on external fan, coat with foil, poke a hole, cold finger

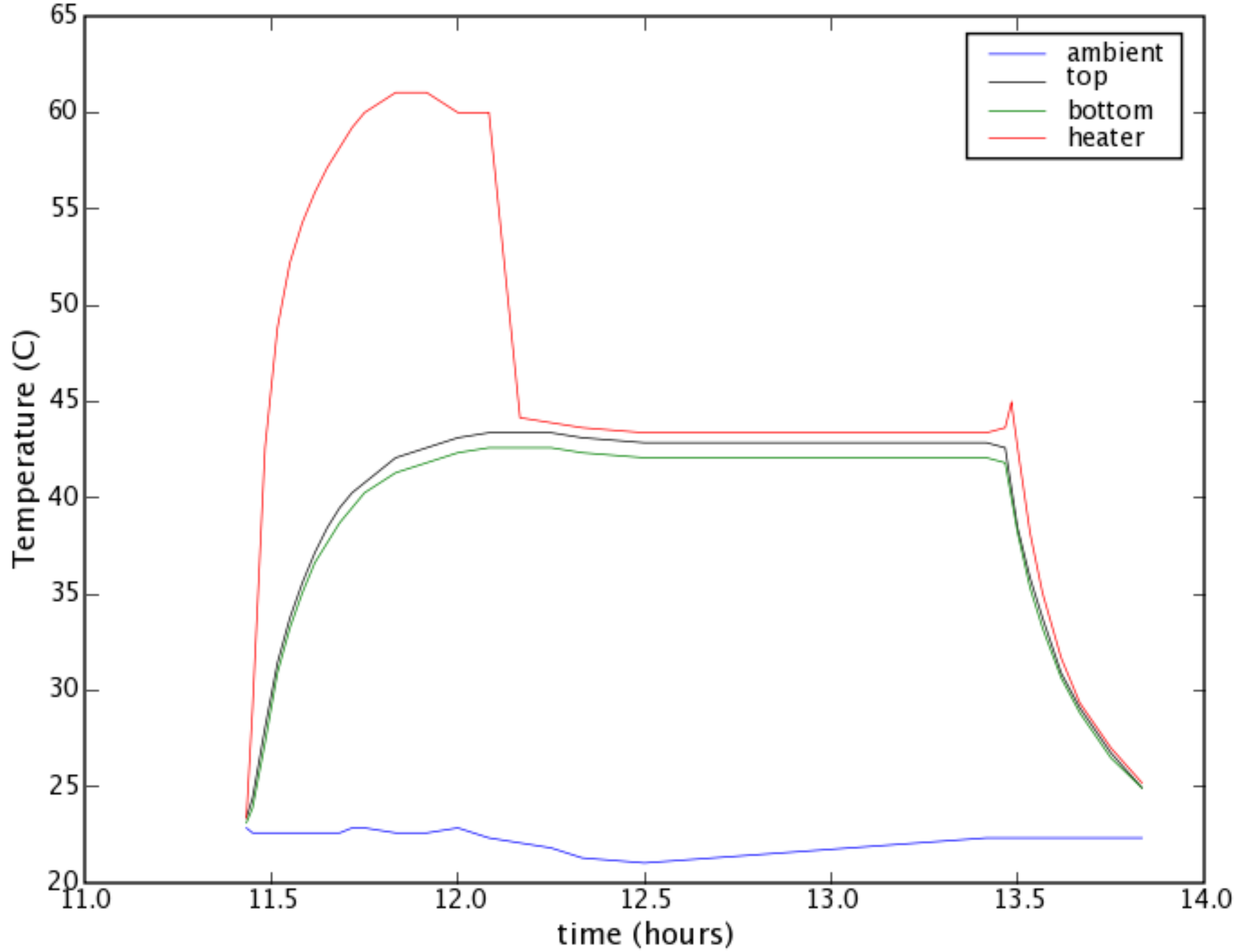
Random Notes, continued

- Send your data points to me via e-mail so I can present the amalgam of results to the class
  - use format:

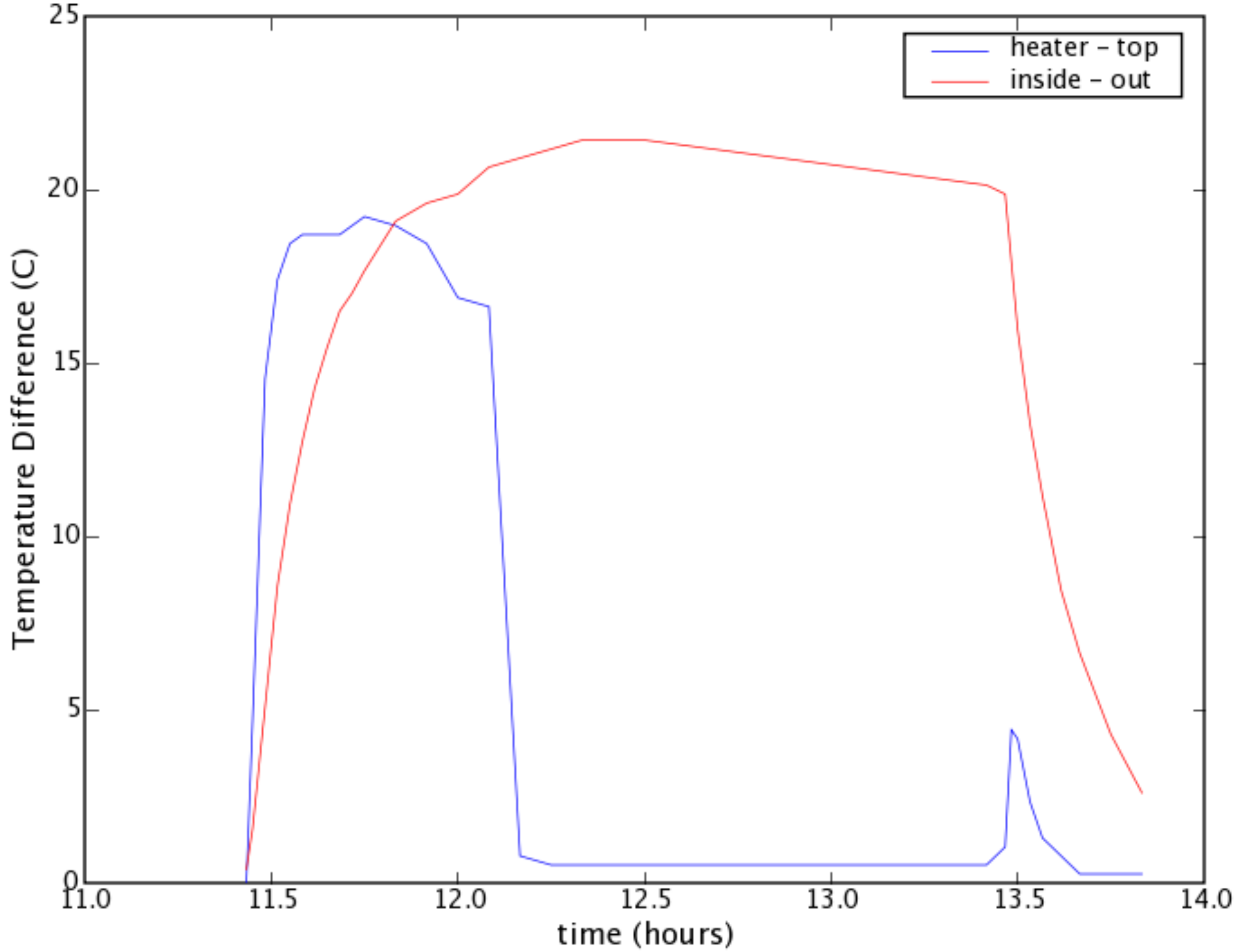
    Δt T_1 T_2 T_3 etc.
  - example:

    165.0 27.6 31.2 32.2 24.8
  - include a description of what each column represents
- Also include basic setup and changes in e-mail so I know what I'm plotting
- Also include in the message temperatures you measure only once, or occasionally (like skin temp.)
- I'll make the data available for all to access for the write-ups

Example Series



Temperature differences



References and Assignment

- Useful text:
  - Introduction to Heat Transfer: Incropera & DeWitt
- Reading in text:
  - Chapter 8 (7 in 3rd ed.) reading assignment: check web page for details

Thermal Building Design

- You can get R-values for common construction materials online
  - see http://www.coloradoenergy.org/procorner/stuff/r-values.htm
- Recall that $R = 5.67 \times t/\kappa$
  - so power, $P = 5.67A\Delta T/R$
- Composite structures (like a wall) get a net R-value
  - some parts have insulation, some parts just studs
  - if we have two areas, A_1 with R_1 and A_2 with R_2, total power is

    $$P = 5.67A_1\Delta T/R_1 + 5.67A_2\Delta T/R_2$$
  - so we can define net R so that it applies to $A_{tot} = A_1 + A_2$
  - $1/R_{tot} = (A_1/A_{tot})/R_1 + (A_2/A_{tot})/R_2$
  - in example on web site, studs take up 15%, rest of wall 85%
  - $P = 5.67A_{tot}\Delta T/R_{tot}$

Handling External Flow as R-value

- On the materials site, they assign *R*-values to the air "layer" up against the walls
  - outside skin $R = 0.17$
  - inside skin $R = 0.68$
- This accounts for both convection *and* radiation. How?
  - recall that power through the walls has to equal the power convected and radiated

    $P = 5.67A(T_{in}\text{-}T_{skin})/R = h_{conv}A(T_{skin}\text{-}T_{out}) + h_{rad}A(T_{skin}\text{-}T_{out})$

    $P = 5.67\text{A}(T_{in}\text{-}T_{skin})/R = h_{eff}A(T_{skin}\text{-}T_{out})$
  - where $h_{rad} \approx 4\sigma\varepsilon T^3$, and $h_{eff} = h_{conv} + h_{rad}$
- We can solve this for T_{skin}, to find

  $T_{skin} = (5.67T_{in}/R + h_{eff}T_{out})/(5.67/R + h_{eff})$

Putting Together

- Inserting the expression for T_{skin} into the conduction piece, we get:

  $P = 5.67A(T_{in}-T_{skin})/R = 5.67A(T_{in}-(5.67T_{in}/R+h_{eff}T_{out})/(5.67/R+h_{eff}))/R$

  – multiply the solitary T_{in} by $(5.67/R+h_{eff})/(5.67/R+h_{eff})$

  – $5.67T_{in}/R$ term cancels out

  $P = 5.67A((h_{eff}T_{in} - h_{eff}T_{out})/(5.67/R+h_{eff}))/R$

  $P = 5.67A(T_{in}-T_{out})\times h_{eff}/(5.67+h_{eff}R)$

  – which now looks like a standard conduction relation between inside and outside temperatures, with an effective R:

  $R_{eff} = R + 5.67/h_{eff}$

- The effective R is the R-value of the original wall plus a piece from the air that looks like $5.67/h_{eff}$
  – the site has interior air layer R_{eff}=0.68, or h_{eff} = 8.3, which is appropriate for radiation plus convection
  – for exterior, they use R_{eff} = 0.17, or h_{eff} = 33, representing windy conditions

A model house

- Ignoring the floor, let's compute the heat load to keep a house some ΔT relative to outside
  - useful to formulate $G = P/\Delta T$ in W/K as property of house
  - Assume approx 40×40 ft floorplan (1600 ft^2)
  - 8 feet tall, 20% windows on wall
  - Wall: 100 m^2, windows: 20 m^2, ceiling: 150 m^2, roof 180 m^2
- Can assess for insulation or not, different window choices, etc.
  - G_{window} = 125, 57, 29 for single, double, or deluxe window
  - G_{wall} = 142, 47 for no insul, insul
  - G_{ceil} = 428, 78 for no insul, insul
  - G_{roof} = 428, 90 for no insul, insul

Dealing with the Ceiling

- The G_{ceil} and G_{roof} require interpretation, since the ΔT across these interfaces is not the full ΔT between inside and outside
  - there is a T_{attic} in between
  - but we know that the heat flow through the ceiling must equal the heat flow through the roof, in equilibrium
  - so $G_{ceil}(T_{in}-T_{attic}) = G_{roof}(T_{attic}-T_{out})$
  - then $T_{attic} = (G_{ceil}T_{in}+G_{roof}T_{out})/(G_{ceil}+G_{roof})$
  - so that $G_{ceil}(T_{in}-T_{attic}) = G_{up}(T_{in}-T_{out})$
  - where $G_{up} = G_{ceil}G_{roof}/(G_{ceil}+G_{roof})$, in effect acting like a parallel combination
- So G_{up} evaluates to:
  - G_{up} = 214, 74, 66, 42 for no/no, ceil/no, no/roof, ceil/roof insulation combinations

All Together Now

- The total power required to stabilize the house is then

  $P_{tot} = G_{tot}\,\Delta T$, where $G_{tot} = G_{window} + G_{wall} + G_{up}$
- For a completely uninsulated house:
  - G_{tot} = 481 W/K
  - requires 7.2 kW to maintain ΔT = 15°C
  - over 5 months (153 days), this is 26493 kWh, costing $2649 at $0.10/kWh
- Completely insulated (walls, ceiling, roof, best windows), get G_{tot} = 118 W/K
  - four times better!
  - save $2000 per cold season (and also save in warm season)