Lecture 21

II

<u>Partial-wave analysis</u>

$$\psi_k(r,\vartheta) = e^{ikz} + f(\vartheta)\frac{e^{ikr}}{r} \quad (r \to \infty)$$

Assume central potential $V(r)$

The radial Schrödinger equation is

$$\left[\frac{1}{r}\frac{d^2}{dr^2}r - \frac{\ell(\ell+1)}{r^2} + k^2 - \frac{2mV}{\hbar^2}\right] R_{k\ell}(r) = 0$$

For $r \to \infty$ $V(\bar{r}) \to 0$

$$R_{k\ell} \sim \frac{1}{kr}\sin\left(kr - \frac{\ell\pi}{2}\right) \quad \left(\text{spherical Bessel function}\right)$$

If $V(r)$ decays faster than $\frac{1}{r}$ the true asymptotic form is

$$R_{k\ell} = \frac{1}{kr}\sin\left(kr - \frac{\ell\pi}{2} + \delta_\ell\right)$$

The general solution is $\psi_k(r,\vartheta) = \sum_{\ell=0}^{\infty} C_\ell R_{k\ell} P_\ell(\cos\vartheta)$

$P_\ell(\cos\vartheta)$ is ℓth Legendre polynomial

Match this form to the asymptotic form

$$e^{ikz} + f(\theta)\frac{e^{ikr}}{r}$$

$$e^{ikz} = \sum_{l=0}^{\infty} i^l (2l+1) j_l(kr) P_l(\cos\theta) \sim$$

$$\sim \sum_{l=0}^{\infty} (2l+1) i^l \frac{\sin(kr - l\pi/2)}{r} P_l(\cos\theta)$$

$$\Downarrow$$

$$\sum_l C_l P_l(\cos\theta) \frac{\sin(kr - l\pi/2 + \delta_l)}{kr} =$$

$$= \sum_l (2l+1) i^l P_l(\cos\theta) \frac{\sin(kr - l\pi/2)}{kr} +$$

$$+ \frac{f(\theta) e^{ikr}}{r}$$

$$C_l = i^l (2l+1) e^{i\delta_l}$$

$$f(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} \frac{C_l}{i^l} \sin\delta_l P_l(\cos\theta)$$

We need to calculate the phase shift δ_l

$$\sigma = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) \sin^2 \delta_l$$

$$f(\theta=0) = \frac{1}{k} \sum_l (2l+1) \cos\delta_l \sin\delta_l + \frac{i}{k} \sum_l (2l+1) \sin^2\delta_l$$

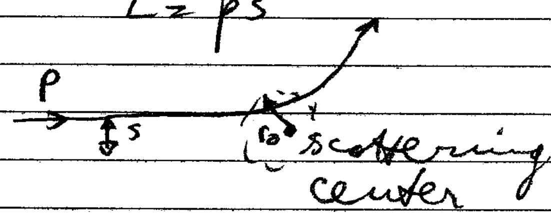
$$\Downarrow$$

$$\sigma = \frac{4\pi}{k} \, \mathrm{Im}\left[f(0)\right]$$

If L angular momentum; p linear momentum

The impact parameter is defined as

$$L = ps$$



r_0 effective radius of interaction

If $s > r_0$ very small interaction

$L = \hbar\sqrt{l(l+1)} \sim \hbar l$; $p = \hbar k$

Interaction is negligible for $l > r_0 k$ i.e. partial waves with $l > r_0 k$ have negligible phase shift.

For low energy scattering $k r_0 \ll 1$ only $l = 0$ scattering $\Rightarrow f(\vartheta) = \frac{1}{k} e^{i\delta_0} \sin\delta_0$ (spherical, independent of ϑ)

$\downarrow$

s-wave scattering