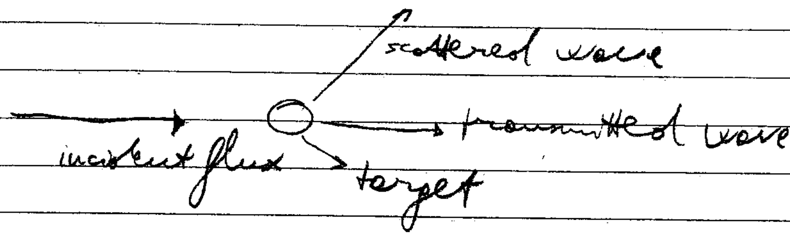
Lecture 20

Elements of scattering theory

Scattering cross section:



Assume the Hamiltonian is

$$H = H_0 + V(\bar{r})$$

$$H_0 \equiv \frac{-\hbar^2}{2m}\nabla^2$$

$V(\bar{r}) \xrightarrow[\bar{r}\to\infty]{} 0$ more rapidly than $\frac{1}{r}$

Before the collision at $t \to -\infty$ a particle can be described by the wave function (along the z direction)

$$\psi_k = e^{ikz}$$

Define two stationary waves

$$\psi_k^{(+)} = e^{ikz} + \text{outgoing wave}$$

$$\psi_k^{(-)} = e^{ikz} + \text{incoming wave}$$

If E is the energy of the particle

$$k = \sqrt{2mE}$$

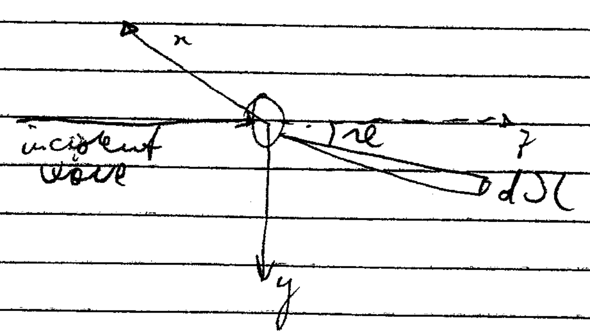
$\psi_k^{(+)}$ is solution to

$$H\psi_k^{(+)} = E\psi_k^{(+)}$$

with boundary condition at $r \to \infty$

$$\psi_k^{(+)} \longrightarrow e^{ikz} + f(\vartheta)\,\frac{e^{ikr}}{r}$$

$f(\vartheta)$ is called <u>scattering amplitude</u>



The number of particles scattered into the vector solid angle $d\vec{\Omega}$ is given by the radial component of the scattered current density

$$J_{sc,r} = \frac{\hbar}{2mi}\left(\left(\psi_k^{(sc)}\right)^* \frac{\partial}{\partial r}\psi_k^{(sc)} - \psi_k^{(sc)}\frac{\partial \psi^{(sc)*}}{\partial r}\right) =$$

$$= \frac{\hbar k}{m r^2}\left|f(\theta)\right|^2$$

($\Psi_{inc} = e^{ikz}$ must have a finite [illegible] in x, y [illegible] cross [illegible] terms.)

Number of particles passing through the surface $d\bar{S}$ (defined by the angle $d\Omega$) per unit time

$$dN = \bar{J}_{sc}\cdot d\bar{S} = r^2\,\bar{J}_{sc}\cdot d\bar{\Omega} = r^2 J_{sc,r}\,d\Omega$$

dN has to be proportional to $\bar{J}_{incident}$

$$dN \propto \bar{J}_{inc} \Rightarrow dN = d\sigma\,\bar{J}_{inc}$$

$d\sigma$ is the differential scattering cross section

The total scattering cross section

$$\sigma = \int d\sigma = \int_{4\pi}\left(\frac{d\sigma}{d\Omega}\right) d\Omega$$

$$r^2 J_{sc,r}\, d\Omega = J_{inc}\, d\sigma$$

$$J_{inc} = \frac{\hbar k}{m} \Rightarrow r^2 J_{sc,r}\, d\Omega = \frac{\hbar k}{m}\, d\sigma$$

$$\Downarrow$$

$$d\sigma = |f(\theta)|^2\, d\Omega$$

Total cross section

$$\sigma = \int d\sigma = 2\pi \int_0^\pi |f(\theta)|^2 \sin\theta\, d\theta$$