Lecture 18

Selection rules

We need to evaluate $\langle \psi_b | \vec{r} | \psi_e \rangle$

Assume ψ_i for H atom

$$\langle n'l'm' | \vec{r} | nlm \rangle$$

Selection rules on m

$$[L_z, x] = i\hbar y \qquad [L_z, y] = -i\hbar x \qquad [L_z, z] = 0$$

$$\langle n'l'm' | [L_z, z] | nlm \rangle = \langle n'l'm' | L_z z - z L_z | nlm \rangle =$$

$$= \langle n'l'm' | [m'\hbar z - z m\hbar] | nlm \rangle =$$

$$= (m' - m)\hbar \langle n'l'm' | z | nlm \rangle = 0$$

$$\Downarrow$$

$$m' = m \quad \text{or} \quad \langle n'l'm' | z | nlm \rangle = 0$$

[i.e. if there is a transition $m' = m$]

(1) $\langle n'l'm' | [L_z, x] | nlm \rangle = \langle n'l'm' | (L_z x - x L_z) | nlm \rangle =$

$$= (m' - m)\hbar \langle n'l'm | x | nlm \rangle = i\hbar \langle n'l'm' | y | nlm \rangle$$

(2) $\langle n'l'm'|[L_z, y]|nlm\rangle = (m'-m)\hbar\langle n'l'm'|y|nlm\rangle =$
$= -i\hbar\langle n'l'm'|x|nlm\rangle$

Combining (1) and (2)

$$(m'-m)^2\langle n'l'm'|x|nlm\rangle = i(m'-m)\langle n'l'm'|y|nlm\rangle =$$
$$= \langle n'l'm'|x|nlm\rangle$$

$$(m'-m)^2 = 1 \quad \text{or} \quad \langle n'l'm'|x|nlm\rangle = \langle n'l'm'|y|nlm\rangle = 0$$

$m'-m = \Delta m = \pm 1\ ; 0$ if not no transitions
[spin of photon $= 1$ $S_z = \pm 1, 0$]

Selection rule for l

$$[L^2, [L^2, \bar{r}]] = 2\hbar^2(\bar{r}L^2 + L^2\bar{r})$$

$$\langle n'l'm'|[L^2, [L^2, \bar{r}]]|nlm\rangle =$$
$$= 2\hbar^2\langle n'l'm'|(\bar{r}L^2 + L^2\bar{r})|nlm\rangle =$$
$$= 2\hbar^4[l(l+1) + l'(l'+1)]\langle n'l'm'|\bar{r}|nlm\rangle =$$

$$= \langle n'l'm'|(L^2[L^2,\bar{r}] - [L^2,\bar{r}]L^2)|nlm\rangle =$$

$$= \hbar^2[l'(l'+1) - l(l+1)]\langle n'l'm'|[L^2,\bar{r}]|nlm\rangle =$$

$$= \hbar^2[l'(l'+1) - l(l+1)]\langle n'l'm'|(L^2\bar{r} - \bar{r}L^2)|nlm\rangle =$$

$$= \hbar^4[l'(l'+1) - l(l+1)]^2\langle n'l'm'|\bar{r}|nlm\rangle$$

$$2[l(l+1) + l'(l'+1)] = [l'(l'+1) - l(l+1)]^2$$

or $\langle n'l'm'|\bar{r}|nlm\rangle = 0$

$$[(l'+l+1)^2 - 1][(l'-l)^2 - 1] = 0$$

$$\Downarrow$$

$$(l'-l) = \Delta l = \pm 1 \qquad [\text{if } l' = l = 0 \Rightarrow \langle n'l'm'|\bar{r}|nlm\rangle = 0$$

photon has spin 1 $\Rightarrow$ addition of angular momenta

$l' = l+1$; $l' = l$ not allowed in dipole radiation ; $l' = l-1$