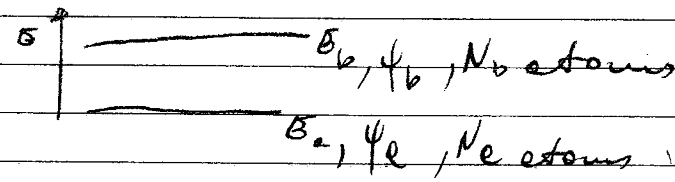
Spontaneous emission



$A \equiv$ spontaneous emission rate from $b \to a$ due to thermal radiation

Stimulated emission

\# particles per unit time from $b \to a$ $\quad N_b R_{b\to a}$

$$R_{b\to a} = \frac{\pi}{3\varepsilon_0 \hbar^2} |\tilde{\vec{p}}|^2 \rho(\omega_0)$$

Absorption rate $\quad R_{a\to b} = \dfrac{\pi}{3\varepsilon_0\hbar^2} |\tilde{\vec{p}}|^2 \rho(\omega_0)$

\# particles per unit time from $a \to b$ $\quad N_a R_{a\to b}$

$$\frac{dN_b}{dt} = \underbrace{-N_b A}_{\text{spont. em.}} \underbrace{- N_b R_{b\to a}}_{\text{stim. em.}} + \underbrace{N_a R_{a\to b}}_{\text{absorption}}$$

In thermal equilibrium $\quad \dfrac{dN_b}{dt} = 0$

$$\rho(\omega_0) = \frac{A}{\left(\dfrac{N_a}{N_b} - 1\right)\dfrac{\pi}{3\varepsilon_0\hbar^2}|\tilde{\vec{p}}|^2}$$

From stat. mech.

$$\frac{N_a}{N_b} = \frac{e^{-E_a/k_BT}}{e^{-E_b/k_BT}} = e^{\frac{\hbar\omega_0}{k_BT}}$$

$$\Downarrow$$

$$\rho(\omega_0) = \frac{A}{\left(e^{\frac{\hbar\omega_0}{k_BT}}-1\right)\frac{\pi|\vec{p}|^2}{3\varepsilon_0\hbar^2}}$$

From planck's blackbody formula the energy density of thermal radiation

$$\rho(\omega) = \frac{\hbar}{\pi^2c^3}\frac{\omega^3}{e^{\hbar\omega/k_BT}-1}$$

$$\Downarrow$$

$$A = \frac{\omega_0^3\hbar}{\pi^2c^3}\frac{\pi|\vec{p}|^2}{3\varepsilon_0\hbar^2} = \frac{\omega_0^3}{3\pi\hbar\varepsilon_0}\frac{|\vec{p}|^2}{c^3}$$

If we are out of equilibrium

In an interval of time dt a fraction Adt of atoms in the upper state will go to the lower state due to spontaneous emission

$$dN_b = -N_b A\,dt$$

$$\Downarrow$$

$$N_b(t) = N_b(0)\,e^{-At}$$

$\tau = \frac{1}{A}$ is the lifetime of the state

For several lower energy states $\psi_1, \psi_2, \psi_3 \ldots$ the transition rates add $\Rightarrow \tau = \dfrac{1}{A_1 + A_2 + A_3 \ldots}$