Solution of Week 6 Assignment (Chapter 12: 12.2, 12.3, 12.14)

In this assignment:

$$\chi^2 = \sum_k \frac{(O_k - E_k)^2}{E_k}$$ where E_k is expected no. and O_k is observed no.

12.2

Bin	Times in bin	Observed no.	Expected no.
1	$t < \bar{t} - \sigma_t$	5	4.8
2	$\bar{t} - \sigma_t \le t < \bar{t}$	9	10.2
3	$\bar{t} \le t < \bar{t} + \sigma_t$	13	10.2
4	$t + \sigma_t \le t$	3	4.8

Bin 1 and Bin 4: Expected no. = 30*[0.5*(1-68.27%)]

Bin 2 and Bin 3: Expected no. = 30*(0.5*68.27%)

$\chi^2 = 1.59$

$\chi^2 = 1.59$ < no. of bin (= 4), therefore the measurement can be regarded as normally distributed.

12.3

Face showing (k)	1	2	3	4	5	6
Observed no.	20	46	35	45	42	52
Expected no.	40	40	40	40	40	40

For each face: Expected no. = 240/6

$\chi^2 = 15.85$ >> n=6, therefore, die is likely to be loaded.

12.14 (a)

x	1	2	3	4	5
y (observed)	60	56	71	66	86
y = 50 + 6x	56	62	68	74	80

$\chi^2 = 10.0625$

12.14 (b)

In this problem, there is no constraint from the observed data (i.e. we do not use any information from the observed data to calculate the expected data), therefore, degrees of freedom = no. of data points = 5.

Therefore, reduced $\chi^2 = \chi^2 / 5 = 2.0125 \Rightarrow \text{Prob}(\chi^2 / 5 > 2.0125) = 7.5\% > 5\%$

Therefore, we should not reject the expected relation on 5% (significant) level.