[Homework 5]

PHYSICS 2BL: HOMEWORK SOLUTIONS, CHAPTER 5

$\boxed{5.12}$

$$G(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-X)^2}{2\sigma^2}}$$

$G(x)_{max}$ is when $x = X$

$$G(x)_{max} = \frac{1}{\sigma\sqrt{2\pi}}\, \underbrace{e^{-\frac{(X-X)^2}{2\sigma^2}}}_{=e^0=1} = \frac{1}{\sigma\sqrt{2\pi}}$$

$$G(x)_{halfmax} = \frac{1}{2\sigma\sqrt{2\pi}} = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-X)^2}{2\sigma^2}}$$

$$\tfrac{1}{2} = e^{-\frac{(x-X)^2}{2\sigma^2}}$$

$$\ln \tfrac{1}{2} = \frac{-(x-X)^2}{2\sigma^2}$$

$$2\sigma^2 \ln 2 = (x-X)^2$$

$\sigma\sqrt{2\ln 2} = x - X =$ half width half max (HWHM)

$2\sigma\sqrt{2\ln 2} = 2(x-X) =$ full width half max (FWHM)

$\boxed{5.20}$ mean height $= 69''$ 1000 men
$\sigma = 2''$

(a) # of men between ~~68~~ 67" and 71"

Probability = 68%, since 1σ

$$.68(1000) = \boxed{680 \text{ men}}$$

(b) # of men > 71"

½ men > 1σ more than mean

$$\frac{1-0.68}{2} \times 1000 = \boxed{160 \text{ men}}$$

(c) # of men > 75"

½ men > 3σ more than mean

$$\frac{1-0.997}{2} \times 1000 = 1.5 \approx \boxed{2 \text{ men}}$$

(d) # of men between 65 and

½(men 2σ more than mean — men σ more than mean)

$$2\sigma : 0.954(1000) = 954$$

$$\sigma = 0.68(1000) = 680$$

$$\frac{954-680}{2} = \boxed{137 \text{ men}}$$

5.22 Prove $\int_{X-t\sigma}^{X+t\sigma} G_{X,\sigma}(x)\,dx = \frac{1}{\sqrt{2\pi}} \int_{-t}^{t} e^{-z^2/2}\,dz$ = Probability (within $t\sigma$)

* $\text{Prob} = \frac{1}{\sigma\sqrt{2\pi}} \int_{X-t\sigma}^{X+t\sigma} e^{-\frac{(x-X)^2}{2\sigma^2}}\,dx$

* Set $\frac{x-X}{\sigma} = z$, so $x - X = \sigma z \longrightarrow dx = \sigma\,dz$

* $x_1 = X + t\sigma$, so $\frac{x_1 - X}{\sigma} = t = z_1$

$x_2 = X - t\sigma$, so $\frac{x_2 - X}{\sigma} = -t = z_2$

* $\text{Prob} = \frac{1}{\sigma\sqrt{2\pi}} \int_{z_2}^{z_1} e^{-z^2/2}\,\sigma\,dz$

$$\boxed{\text{Prob} = \frac{1}{\sqrt{2\pi}} \int_{-t}^{t} e^{-z^2/2}\,dz}$$

5.35 (a) Student's value is 3σ from accepted value.
Probability of getting answer ≥ 3σ is 1 − 0.997 = 0.3%

(b) Student's results strongly suggest a systematic error.