Physics 2BL HW#2 : 4.5, 4.13, 4.18, 4.20, 4.23

4.5.) prove $\sum[(x_i-\bar{x})^2] = \sum[(x_i)^2] - \frac{1}{N}[\sum(x_i)^2]$

a) expand $(x_i-\bar{x})^2$: $(x_i-\bar{x})^2 = x_i^2 - 2x_i\bar{x} + \bar{x}^2$

so $\sum_i[(x_i-\bar{x})^2] = \sum[x_i^2] - \sum[2x_i\bar{x}] + \sum[\bar{x}^2]$

$\sum[2x_i\bar{x}] = \sum[2x_i(\frac{1}{N}\sum x_i)]$ (by def. of $\bar{x}$)

$= \frac{2}{N}\sum(x_i \sum x_i)$ (let $\sum x_i$ = no)

$= \frac{2}{N}(\sum x_i)^2$

$\sum[\bar{x}^2] = \sum(x_i-\bar{x})^2$ when $x_i = 0$

$= (x_1-\bar{x})^2 + (x_2-\bar{x})^2 + (x_3-\bar{x})^2 + \dots (x_N-\bar{x})^2$

$= N\bar{x}^2 = N(\frac{1}{N}\sum x_i)^2 = \frac{1}{N}(\sum x_i)^2$

so $\sum_i[(x_i-\bar{x})^2] = \sum[x_i^2] - \frac{2}{N}(\sum x_i)^2 + \frac{1}{N}[\sum x_i]^2$

$= \sum[x_i^2] - \frac{1}{N}[\sum x_i]^2$.

//

b) Data : 0.11s, 0.13 s, 0.12 s

$\bar{x} = \frac{0.11 + 0.13 + 0.12}{3} = 0.12$ s

$\sum(x_i-\bar{x})^2 = (0.11-0.12)^2 + (0.13-0.12)^2 + (0.12-0.12)^2 = 0.0002$

$\sum(x_i)^2 = 0.11^2 + 0.13^2 + 0.12^2 = 0.0434$

$(\sum[x_i])^2 = 0.1296$

so $0.0434 - \frac{1}{3}\cdot(0.1296) = 0.0434 - 0.0432 = 0.0002$. //

Problem 4.13

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MEAN = 14.93

Standard Deviation = 3.93

a.) mean = 8.15 and standard deviation = 0.4

b.) we expect 68% of 30 to be within the first standard deviation or about 20.
there are actually 22 that lie within the first standard deviation.

c.) we expect 95% of 30 or about 28-29 to lie within the range 8.07 and 8.23.
there are actually 28 that lie in this range.

Problem 4.18

$\sigma_u = 10$ m/s $\qquad \sigma_{\bar{x}} = \frac{\sigma_u}{\sqrt{N}}$

if $\sigma_{\bar{x}} = \pm 3 \rightarrow \frac{10}{3} = \sqrt{N} \rightarrow N = 3.33^2$ or ≈ 11

if $\sigma_{\bar{x}} = \pm 0.5 \rightarrow \frac{10}{0.5} = \sqrt{N} \rightarrow N = 20^2$ or $400!$

Problem 4.20

Mass m (kg)	0.513	0.581	0.634	0.691	0.752	0.834	0.901	0.95
Period T (s)	1.24	1.33	1.36	1.44	1.5	1.59	1.65	1.69
$k = 4\pi^2 m/T^2$	13.17	12.97	**13.532**	**13.156**	**13.19**	**13.024**	**13.065**	**13.13**

mean k = **13.16**

SDOM = **0.171**

Problem 4.23

Basically, the question is how does a 0.4% error in η change e? From equation 3.26 we see that $\delta q/|q| = |n|(\delta x/|x|)$ if $q = x^n$

so $\delta e/e = (3/2)\delta\eta/\eta = 0.6\%$