Rejection of Data

Rejecting data in an unwarranted fasion can bias our measurements. Whether or not to reject data clearly depends on the quality of the measurement.

If there is suspision of a measurement error, data should be rejected without looking at the value of the measurment.

We develop a presciption for data rejection based on the value of the measuement for errors which are gaussian. This prescription is called Chauvenet's criterion.

Data are rejected if we expect less than 0.5 measurements with a deviation from the mean as large or larger than the one in question.

If one datum is rejected, we recompute the mean and recheck the remaining data.

Example: Chauvenet's Criterion

A student makes 14 measurements of the period of a damped oscillator, with the following results in seconds:

0.7, 0.3, 0.9, 0.3, 0.6, 0.9, 0.8, 0.7, 0.8, 1.2, 0.5, 0.9, 0.9, 0.3

Should any of these measurements be dropped in taking the average (according to Chauvenet)?

- **Take the average of all 14 gives: 0.70**
- **Sigma is: 0.27**
- **1.2 is furthest off mean. t=0.50/0.27 (1.85 sigma off).**
- **Prob. of one event being futher off is 6.43%.**
- **We expect 14(0.0643)=0.9 events further off.**
- **We should not drop this (or any) of the measurements.**

Weighted Averages

Suppose 2 students measure *x*.
Student A gets: $x = x_A \pm \sigma_A$
Student B gets: $x = x_B \pm \sigma_B$

To get the best estimate of the true value of *x*, take a weighted average.

$$x_{\text{best}} = \bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

$$\sigma^2_{x_{\text{best}}} = \frac{1}{\sum_{i=1}^{n} w_i}$$

$$w_i = \frac{1}{\sigma_i^2}$$

We can use the principle of maximum likelihood to derive these weighted average formulas. This assumes the measurements follow the normal distribution. The likelihood is the product of probabilities.

$$L = Ce^{-\sum_{i=1}^{n} \frac{(x_i - X)^2}{2\sigma_i^2}} = Ce^{\frac{-\chi^2}{2}}$$

To maximize the likelihood, we must minimize the χ^2.

$$\chi^2 = \sum_{i=1}^{n} \frac{(x_i - X)^2}{\sigma_i^2}$$

$$\frac{d\chi^2}{dX} = 0$$

Example: Weighted Average

Three measurements of g are made:

- **$g = 9.9 \pm 0.1$ $\quad w=100$**
- **$g = 9.8 \pm 0.2$ $\quad w=25$**
- **$g = 9.7 \pm 0.5$ $\quad w=4$**

$g = (990 + 245 + 38.8)/129 = 9.87 \pm 0.09$