

Measure the Mean Density of the Earth

- Measure earth's radius (curvature) plus local g.
- Use Newtons constant $G=6.67 \times 10^{-11}$.
- Calculate average density ρ and determine which element(s) constitute the major portion of the earth.
Aim for error on ρ of 1% to 10%.



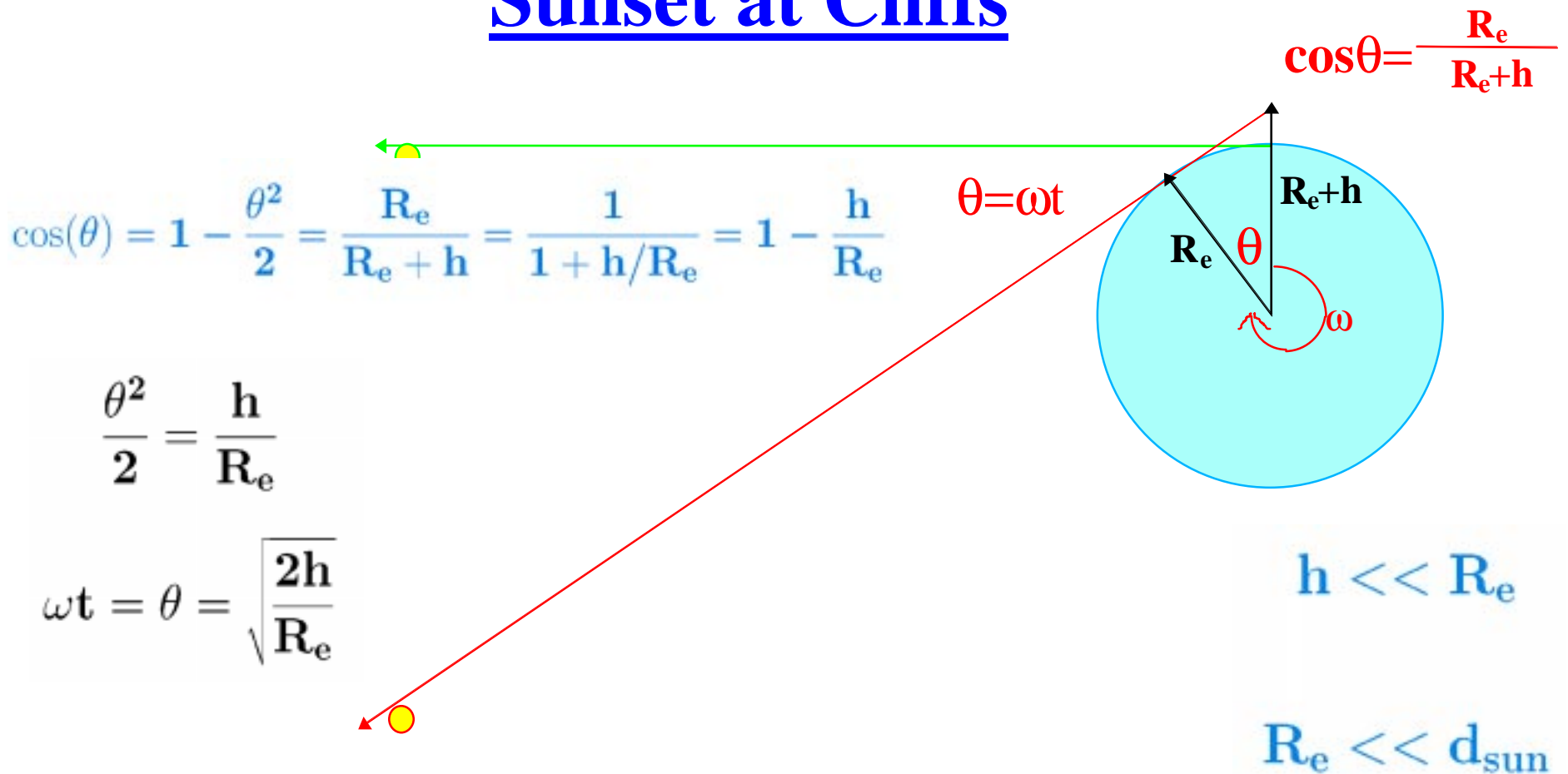
rock	2.7
aluminum	2.7
zinc	7.14
iron	7.20
nickel	8.85
copper	8.89
silver	10.5
lead	11.34
mercury	13.60
gold	19.3

$$F = \frac{GMm}{r^2}$$

$$g = \frac{F}{m} = \frac{GM}{r^2} = \frac{G(\frac{4}{3}\pi R_e^3)\rho}{R_e^2}$$

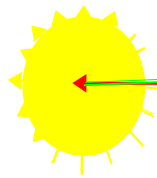
$$\rho = \frac{3}{4\pi} \frac{g}{GR_e}$$

Measure Radius of Earth from Δt Sunset at Cliffs

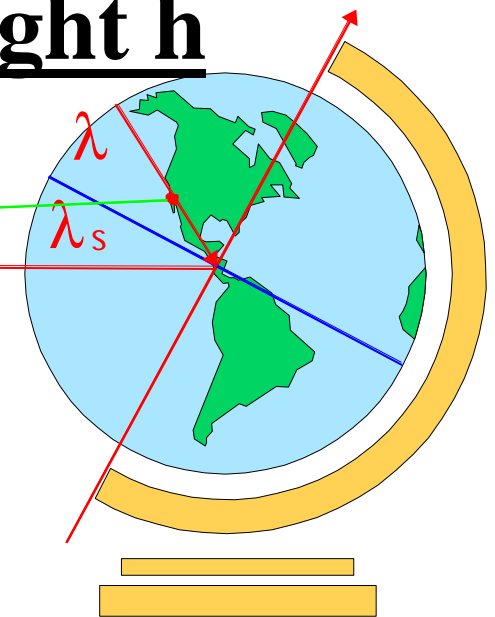


$$t = \frac{1}{\omega} \sqrt{\frac{2h}{R_e}} = \frac{(24)(60)(60)\text{sec}}{2\pi} \sqrt{\frac{2(50\text{m})}{6,000,000\text{m}}} = 50\text{sec}$$

Time Delay of Sunset at Height h



winter view



$$\omega_e t = \sqrt{\frac{2h}{R_e(\cos^2(\lambda) \cos^2(\lambda_s) - \sin^2(\lambda) \sin^2(\lambda_s))}}$$

$$\Delta t = t(h_1) - t(h_2) = \frac{1}{\omega_e} \sqrt{C} (\sqrt{h_1} - \sqrt{h_2}) \frac{1}{\sqrt{R_e}}$$

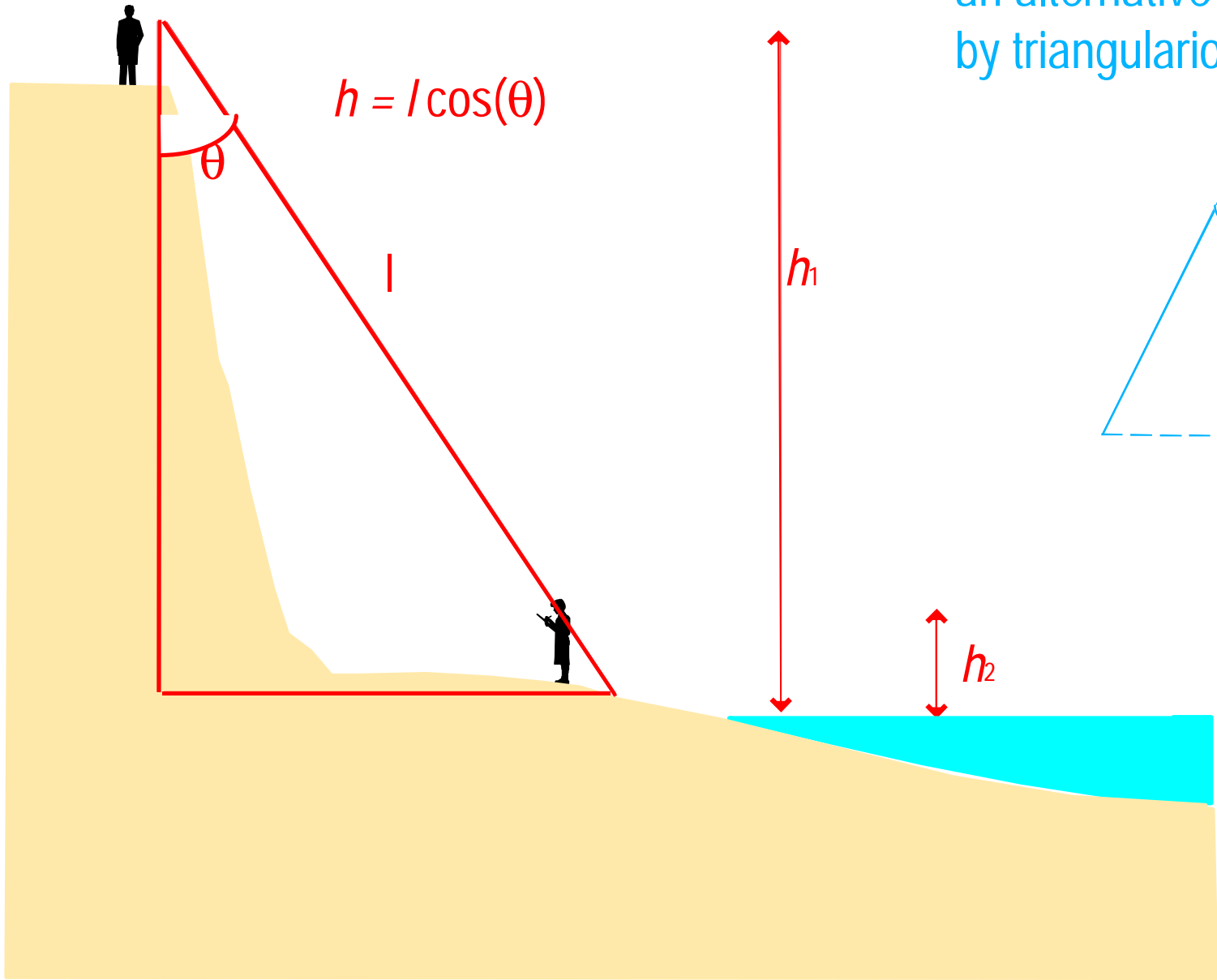
$$C = \frac{2}{(\cos^2(\lambda) \cos^2(\lambda_s) - \sin^2(\lambda) \sin^2(\lambda_s))}$$

$$\lambda = 32.87^\circ$$

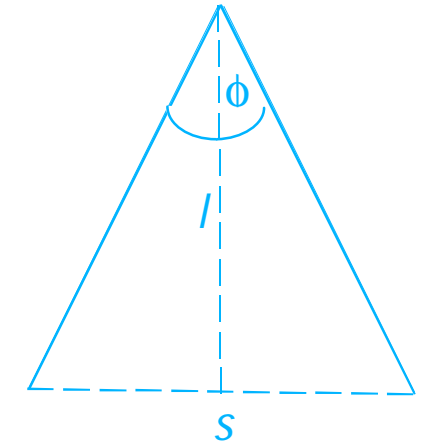
$$R_e = \frac{C}{(\omega_e \Delta t)^2} (\sqrt{h_1} - \sqrt{h_2})^2$$

$$\lambda_s = -23.4 \sin(2\pi d/365)$$

**d=days since Sept. 22
(or March 20).**



an alternative way to get l is by triangulation



pendulum

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$\rho = \frac{3g}{4\pi GR_e}$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta g}{g} + \frac{\Delta R_e}{R_e}$$

Measuring the Height of the Cliff

**We need to know the error on our measurements
if they are to be useful.**

- **Random Errors can be estimated and reduced by repeated measurements. They can also be estimated by using sensible arguments (1/2 the finest scale reading...). Statistical (random) errors are well studied.**
- **Systematic Errors are harder to estimate and reduce. They come from miscalibrations, neglecting (small) corrections, mistakes.**
- **For either type of error, we need to know how to propagate our errors on a measured quantity into errors on a derived quantity.**

Error Propagation

Measure x, y, z and derive quantity q and its error δq .

$$q = x + y + z$$

$$\delta q = \delta x + \delta y + \delta z$$

don't know sign of δx .

$$\delta q = |\delta x| + |\delta y| + |\delta z|$$

worst case.

$$\delta q = \sqrt{|\delta x|^2 + |\delta y|^2 + |\delta z|^2}$$

independent random errors.

$$q = xy$$

$$q + \delta q = (x + \delta x)(y + \delta y) = xy + x\delta y + y\delta x$$

$$\delta q = x\delta y + y\delta x$$

$$\delta q = \sqrt{(x\delta y)^2 + (y\delta x)^2}$$

Summary of Error Propagation

For $q = q(x, y)$,
$$\sigma_q = \sqrt{\left(\frac{\partial q}{\partial x} \sigma_x\right)^2 + \left(\frac{\partial q}{\partial y} \sigma_y\right)^2}$$

where σ_x is the RMS error on x (standard deviation)...

For product ($q=xy$) and ratio ($q=x/y$), fractional errors work

because $\left|\frac{\partial q}{\partial y}\right| = \frac{q}{y}$ implies that
$$\frac{\sigma_q}{q} = \sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2}$$

For higher powers, fractional errors don't work simply. $V = \frac{4}{3}\pi r^3$
gives $\frac{\sigma_V}{V} = 3\frac{\sigma_r}{r}$.

Angles in trig functions must be in radians for error propagation.

We can also propagate errors using a calculator by changing x (then y) by 1 sigma and computing change in q .

More Examples of Error Propagation

Powers

$$q(x, y) = \frac{x^n}{y}$$

$$\delta q = \sqrt{\left(\frac{nx^{n-1}}{y}\delta x\right)^2 + \left(-\frac{x^n}{y^2}\delta y\right)^2}$$

$$\frac{\delta q}{q} = \sqrt{\left(n\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2}$$

Trig. Functions

$$h = l \cos(\theta)$$

$$\delta h = \sqrt{(\cos(\theta)\delta l)^2 + (l \sin(\theta)\delta\theta)^2}$$

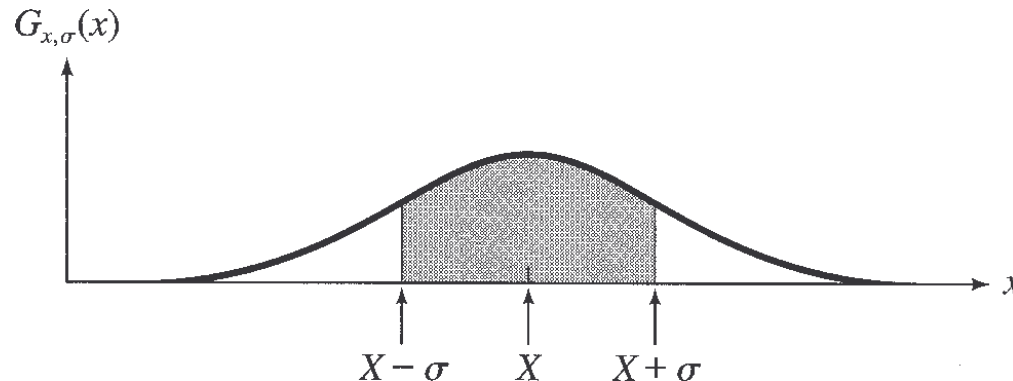
Exp. 1 Function

$$R_e = \frac{C}{(\omega_e \Delta t)^2} (\sqrt{h_1} - \sqrt{h_2})^2$$

$$\delta R = \sqrt{\left(2R\frac{\delta(\Delta t)}{\Delta t}\right)^2 + \left(\frac{C}{(\omega_e \Delta t)^2}(\sqrt{h_1} - \sqrt{h_2})\delta h_1\right)^2 + \left(\frac{C}{(\omega_e \Delta t)^2}(\sqrt{h_1} - \sqrt{h_2})\delta h_2\right)^2}$$

Averaging Data

- Random errors can be reduced by repeated measurements while systematic errors usually cannot.



- The best estimate of the true value of a quantity to be measured is the average.

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

- We can also estimate sigma.

$$\sigma_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$

- The error on the mean decreases with the square root of the number of measurements.

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$$

$$\sigma_x^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 = \overline{(x - \bar{x})^2}$$

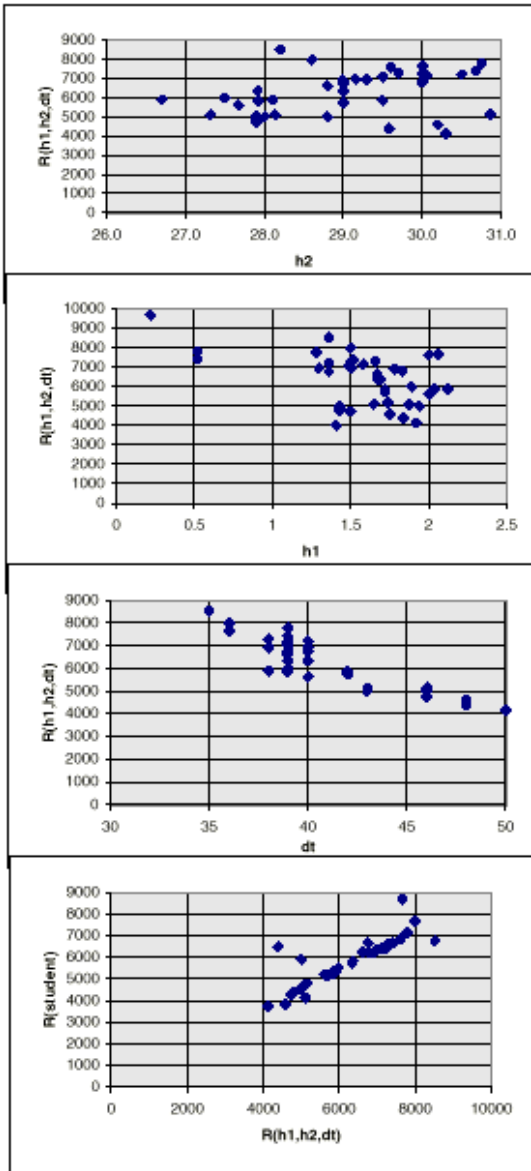
$$\sigma_x^2 = \overline{(x^2 - 2x\bar{x} + \bar{x}^2)} = \bar{x}^2 - 2\bar{x}^2 + \bar{x}^2 = \bar{x}^2 - \bar{x}^2$$

Example: Average R from 38 Students

Physics 2BL

Data Summary

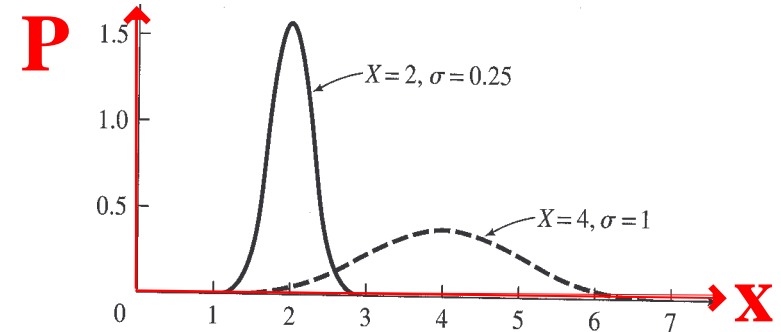
#	Δt	h_1	h_2	R_{form}	R_{stud}
1	50	1.92	30.3	4124	3756
2	48	1.84	29.6	4396	6495
3	48	1.75	30.2	4592	3800
4	46	1.5	27.9	4728	4244
5	46	1.43	27.9	4796	4410
6	43	1.94	28.0	4996	5900
7	46	1.43	28.8	4996	4501
8	43	1.88	27.9	5027	4620
9	43	1.88	28.1	5086	4690
10	43	1.65	27.3	5106	4118
11	46	1.74	30.9	5154	4764
12	40	2	27.7	5618	5190
13	42	1.72	29.0	5717	5132
14	42	1.72	29.5	5848	5252
15	39	2.12	27.9	5858	5270
16	38	2.04	26.7	5884	5400
17	39	2.12	28.1	5907	5311
18	39	1.89	27.5	5982	5523
19	40	1.69	29.0	6339	5700
20	39	1.68	27.9	6351	5790
21	39	1.67	28.8	6633	6230
22	40	1.36	29.0	6761	6660
23	39	1.83	30.0	6797	6200
24	38	1.78	29.0	6907	6200
25	40	1.3	29.3	6934	6200
26	39	1.5	29.2	6965	6338
27	39	1.49	29.5	7084	6370
28	39	1.58	30.1	7135	6420
29	40	1.36	30.5	7209	6432
30	40	1.36	30.5	7209	6455
31	39	1.5	30.0	7225	6422
32	38	1.66	29.7	7288	6624
33	39	1.52	30.7	7409	6680
34	36	2	29.6	7602	6826
35	36	2.06	30.0	7661	8668
36	39	1.28	30.8	7784	7100
37	36	1.5	28.6	7972	7660
38	35	1.36	28.2	8520	6770
x	43	1.41	21.8	3994	4847
x	43	0.52	30.0	7421	8263
x	42	0.52	30.0	7778	8940
x	38	0.22	27.7	9657	8606
ave	41	1.69	29	6191	5766
RMS	3.5	0.24	1.1	1090	1088
actual				6370	6370



- $R=6191$ km.
- True R is 6370.
- $\sigma=1090$ km for each measurement.
- Error on mean should be 167 km (using 38 independent measurements).
- True deviation from expectation is 179 km. (1.07 σ off) (C.L.=28%)
- There does not seem to be a common systematic error which is dominant.

Standard Normal (Gaussian) Distribution

$$P_{X,\sigma}(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-X)^2/2\sigma^2}$$



X and σ are parameters of the Standard Normal Distribution.

X is the true mean of the distribution. (true value)

σ is the RMS width of the distribution. (measurement error)

x is the independent variable. (measured value)

P is the probability (of measuring x).

Central Limit Theorem: When we combine many measurements or sources of error, all probability distributions approach a Gaussian.

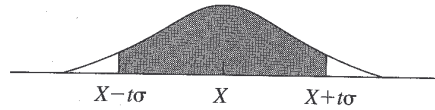
$$\int_{-\infty}^{\infty} P(x) dx = 1$$

$$\int_{-\infty}^{\infty} xP(x) dx = X$$

$$\int_{-\infty}^{\infty} (x - X)^2 P(x) dx = \sigma^2$$

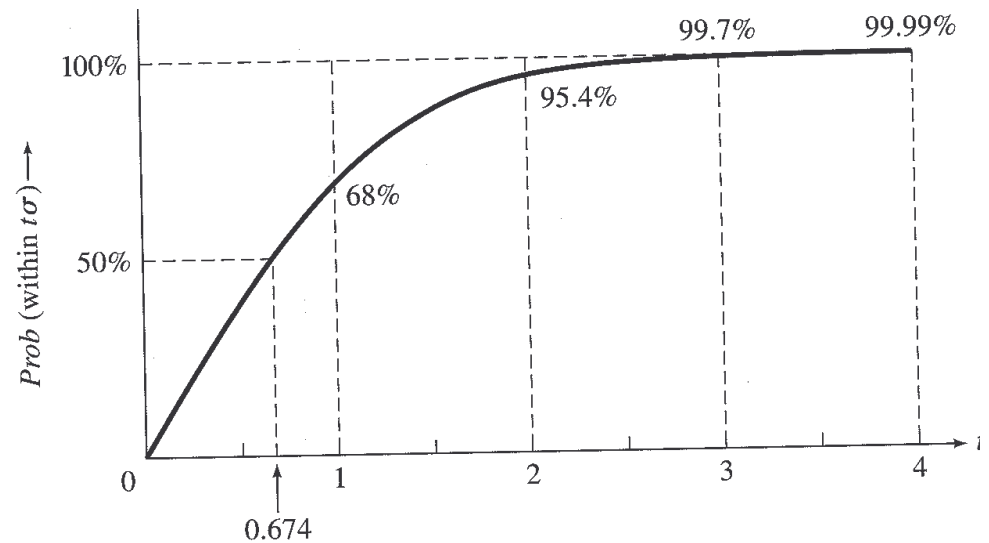
Probabilities in the Normal Distribution

Table A. The percentage probability, $Prob(\text{within } t\sigma) = \int_{X-t\sigma}^{X+t\sigma} G_{X,\sigma}(x) dx$, as a function of t .



t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00	0.80	1.60	2.39	3.19	3.99	4.78	5.58	6.38	7.17
0.1	7.97	8.76	9.55	10.34	11.13	11.92	12.71	13.50	14.28	15.07
0.2	15.85	16.63	17.41	18.19	18.97	19.74	20.51	21.28	22.05	22.82
0.3	23.58	24.34	25.10	25.86	26.61	27.37	28.12	28.86	29.61	30.35
0.4	31.08	31.82	32.55	33.28	34.01	34.73	35.45	36.16	36.88	37.59
0.5	38.29	38.99	39.69	40.39	41.08	41.77	42.45	43.13	43.81	44.48
0.6	45.15	45.81	46.47	47.13	47.78	48.43	49.07	49.71	50.35	50.98
0.7	51.61	52.23	52.85	53.46	54.07	54.67	55.27	55.87	56.46	57.05
0.8	57.63	58.21	58.78	59.35	59.91	60.47	61.02	61.57	62.11	62.65
0.9	63.19	63.72	64.24	64.76	65.28	65.79	66.29	66.80	67.29	67.78
1.0	68.27	68.75	69.23	69.70	70.17	70.63	71.09	71.54	71.99	72.43
1.1	72.87	73.30	73.73	74.15	74.57	74.99	75.40	75.80	76.20	76.60
1.2	76.99	77.37	77.75	78.13	78.50	78.87	79.23	79.59	79.95	80.29
1.3	80.64	80.98	81.32	81.65	81.98	82.30	82.62	82.93	83.24	83.55
1.4	83.85	84.15	84.44	84.73	85.01	85.29	85.57	85.84	86.11	86.38
1.5	86.64	86.90	87.15	87.40	87.64	87.89	88.12	88.36	88.59	88.82
1.6	89.04	89.26	89.48	89.69	89.90	90.11	90.31	90.51	90.70	90.90
1.7	91.09	91.27	91.46	91.64	91.81	91.99	92.16	92.33	92.49	92.65
1.8	92.81	92.97	93.12	93.28	93.42	93.57	93.71	93.85	93.99	94.12
1.9	94.26	94.39	94.51	94.64	94.76	94.88	95.00	95.12	95.23	95.34
2.0	95.45	95.56	95.66	95.76	95.86	95.96	96.06	96.15	96.25	96.34
2.1	96.43	96.51	96.60	96.68	96.76	96.84	96.92	97.00	97.07	97.15
2.2	97.22	97.29	97.36	97.43	97.49	97.56	97.62	97.68	97.74	97.80
2.3	97.86	97.91	97.97	98.02	98.07	98.12	98.17	98.22	98.27	98.32
2.4	98.36	98.40	98.45	98.49	98.53	98.57	98.61	98.65	98.69	98.72
2.5	98.76	98.79	98.83	98.86	98.89	98.92	98.95	98.98	99.01	99.04
2.6	99.07	99.09	99.12	99.15	99.17	99.20	99.22	99.24	99.26	99.29
2.7	99.31	99.33	99.35	99.37	99.39	99.40	99.42	99.44	99.46	99.47
2.8	99.49	99.50	99.52	99.53	99.55	99.56	99.58	99.59	99.60	99.61
2.9	99.63	99.64	99.65	99.66	99.67	99.68	99.69	99.70	99.71	99.72
3.0	99.73									
3.5	99.95									
4.0	99.994									
4.5	99.9993									
5.0	99.99994									

- Probability to lie within plus or minus t standard deviations of the mean is tabulated.
- It may be called the Prob function on your calculator.
- We will use this for rejection of data and calculation of confidence levels.



Example: Confidence Level

Two students measure the Radius of the planet. Student A gets **R=9000 km** and estimates an error of **$\sigma=600$ km**.

Student B get **R=6000 km** and estimates an error of **1000 km**.

Whis the probability that the two measurements that the two measurements would disagree by more than this?

$$q = R_1 - R_2 = 3000km$$

$$\sigma_q = \sqrt{\sigma_1^2 + \sigma_2^2} = 1170km$$

We are 2.56 standard deviations off. We can look up how unlikely that is (Confidence Level) in the table. 98.95% of measurements should be closer than these. C.L. = 1.05%.

The Principle of Maximum Likelihood

$$L = P(x_1)P(x_2)\dots P(x_n) = \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{n}{2}} e^{-\sum_{i=1}^n \frac{(x_i - X)^2}{2\sigma^2}}$$

We can choose the best estimate of X by maximizing L .

At max, $\frac{\partial L}{\partial X} = 0$ which gives $X = \bar{x}$.

We can also derive the addition of errors in quadrature but this is a more sophisticated derivation.

We derive the error on the mean simply by propagating errors.

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \longrightarrow \sigma_{\bar{x}} = \sqrt{\left(\frac{\partial \bar{x}}{\partial x_1} \sigma_{x_1}\right)^2 + \left(\frac{\partial \bar{x}}{\partial x_2} \sigma_{x_2}\right)^2 + \dots + \left(\frac{\partial \bar{x}}{\partial x_n} \sigma_{x_n}\right)^2}$$
$$\longrightarrow \sigma_{\bar{x}} = \sqrt{n \left(\frac{1}{n} \sigma_x\right)^2} = \frac{\sigma_x}{\sqrt{n}}$$

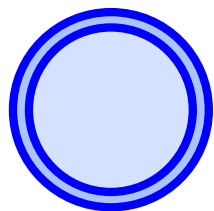
Experiment 2

- 1) You are to devise a simple, fast, and non-destructive method to measure the variation in thickness of the shell of a large numbers of the balls in each shipment arriving at a number of stores, to determine if the variation in thickness is much less than 10%.
- 2) You are to devise a method to measure the thickness and density of the outer and inner cylinders without damaging them so that rods not within specifications will not be used in the machine.

Both problems can be solved by measuring the moment of inertia and mass of the objects.

In (1) we need speed and only need to measure variation in thickness. We can roll the balls down a ramp.

In (2) we need to measure absolute thickness and density but do not need to process many objects. We can use a torsion pendulum.

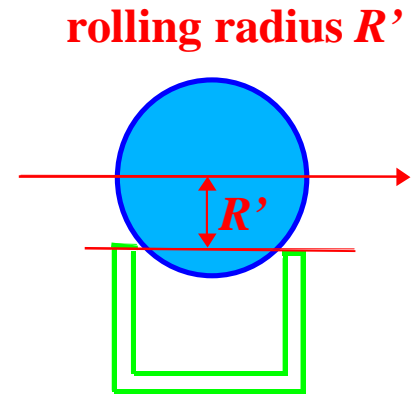
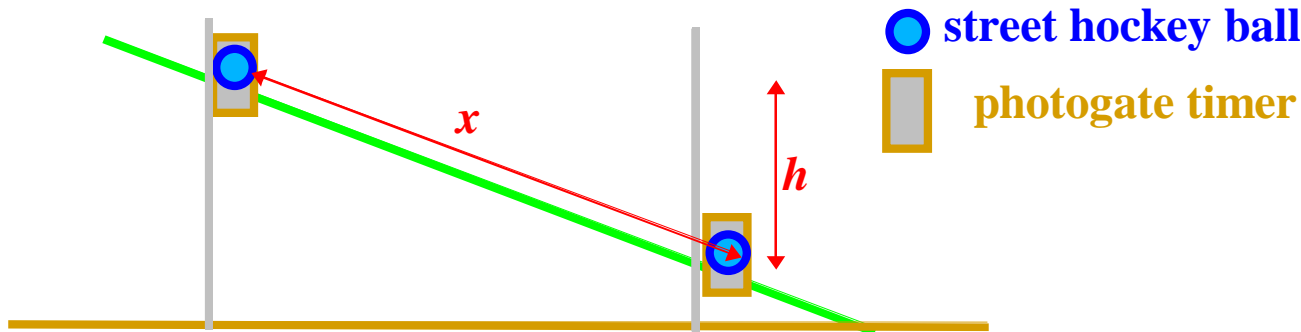


$$I = \frac{2}{5}M \frac{R^5 - r^5}{R^3 - r^3}$$

$$I = \frac{mr^2}{2} + \frac{M(R^2 + r^2)}{2}$$



The Rolling Ball



$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$v = R'\omega$$

$$Mgh = \frac{1}{2}v^2 \left(M + \frac{I}{R'^2} \right)$$

$$v = \sqrt{\frac{2Mgh}{M + \frac{I}{R'^2}}}$$

$$v = \frac{2x}{t} = \sqrt{\frac{2gh}{1 + \frac{2}{5} \frac{R'^5 - r^5}{R'^2(R'^3 - r^3)}}$$

- We must measure many balls of each type so that we can experimentally determine the spread in thickness for each type. Plot histograms.
- Determine thickness for error estimates.
- Compare expected error on thickness to measured RMS.
- Is initial velocity and position reproducible?
- Compute dr/dt numerically on calculator.

The Cylinder and the Torsion Pendulum

- We want an absolute measurement of the cylinder radii and densities.
- Measure R , r , T , and total M .
- Calibrate restoring torque constant κ by using a solid cylinder for which I can be calculated.
- Minimizing wobble of cylinder is most critical experimental problem.

$$N = -\kappa\theta = I\ddot{\theta}$$

$$\omega = \dot{\theta}$$

$$T = 2\pi \sqrt{I/\kappa}$$

$$\frac{M}{M_{tot}} = \frac{I_{tot}}{\frac{1}{2}M_{tot}R^2} - \left(\frac{r}{R}\right)^2$$

Rejection of Data

Rejecting data in an unwarranted fashion can bias our measurements. Whether or not to reject data clearly depends on the quality of the measurement.

If there is suspicion of a measurement error, data should be rejected without looking at the value of the measurement.

We develop a prescription for data rejection based on the value of the measurement for errors which are gaussian. This prescription is called Chauvenet's criterion.

Data are rejected if we expect less than 0.5 measurements with a deviation from the mean as large or larger than the one in question.

If one datum is rejected, we recompute the mean and recheck the remaining data.

Example: Chauvenet's Criterion

A student makes 14 measurements of the period of a damped oscillator, with the following results in seconds:

0.7, 0.3, 0.9, 0.3, 0.6, 0.9, 0.8, 0.7, 0.8, 1.2, 0.5, 0.9, 0.9, 0.3

Should any of these measurements be dropped in taking the average (according to Chauvenet)?

- Take the average of all 14 gives: **0.70**
- Sigma is: **0.27**
- 1.2 is furthest off mean. $t=0.50/0.27$ (1.85 sigma off).
- Prob. of one event being further off is 6.43%.
- We expect $14(0.0643)=0.9$ events further off.
- We should not drop this (or any) of the measurements.

Weighted Averages

Suppose 2 students measure x .

Student A gets: $x = x_A \pm \sigma_A$

Student B gets: $x = x_B \pm \sigma_B$

To get the best estimate of the true value of x , take a weighted average.

$$x_{\text{best}} = \bar{X} = \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i}$$

$$\sigma_{x_{\text{best}}}^2 = \frac{1}{\sum_{i=1}^n w_i}$$

$$w_i = \frac{1}{\sigma_i^2}$$

We can use the principle of maximum likelihood to derive these weighted average formulas. This assumes the measurements follow the normal distribution. The likelihood is the product of probabilities.

$$L = C e^{-\sum_{i=1}^n \frac{(x_i - X)^2}{2\sigma_i^2}} = C e^{-\frac{\chi^2}{2}}$$

To maximize the likelihood, we must minimize the χ^2 .

$$\chi^2 = \sum_{i=1}^n \frac{(x_i - X)^2}{\sigma_i^2}$$

$$\frac{d\chi^2}{dX} = 0$$

Example: Weighted Average

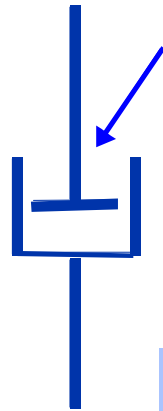
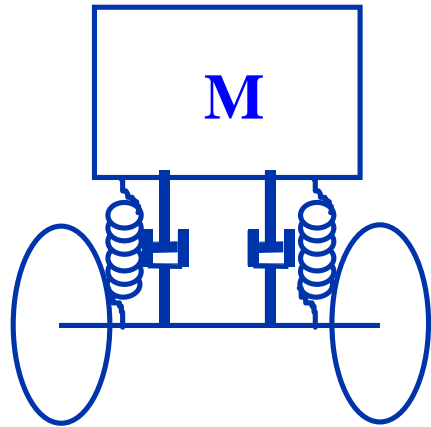
Three measurements of g are made:

- $g = 9.9 \pm 0.1$ $w=100$
- $g = 9.8 \pm 0.2$ $w=25$
- $g = 9.7 \pm 0.5$ $w=4$

$$g = (990 + 245 + 38.8)/129 = 9.87 \pm 0.09$$

Experiment 3

Construct and test a critical damping system for a spring.



Shock absorber damps oscillations of springs. If overdamped, hard jolts transmitted to mass and recovery is very slow. If underdamped, many oscillations of spring. If damping is just right, we call it critically damped and the return to equilibrium is fastest.

spring constant damping coefficient

$$F = -mg - k(y - y_0) - bv$$

equilibrium position = 0

$$ma = -k\left(y - \left[y_0 - \frac{mg}{k}\right]\right) - bv$$

$$\omega_0 = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

damped oscillator frequency

underdamped solution

$$y = y_0 e^{-\frac{b}{2m}t} \cos(\omega_0 t)$$

overdamped solution

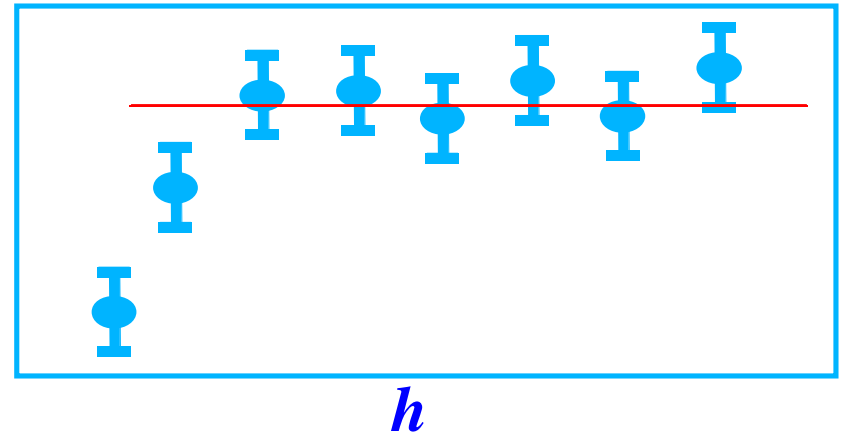
$$y = y_0 e^{-\left(\frac{b}{2m} \pm \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}\right)t}$$

for critical damping both of the above reduce to a simple exponential when

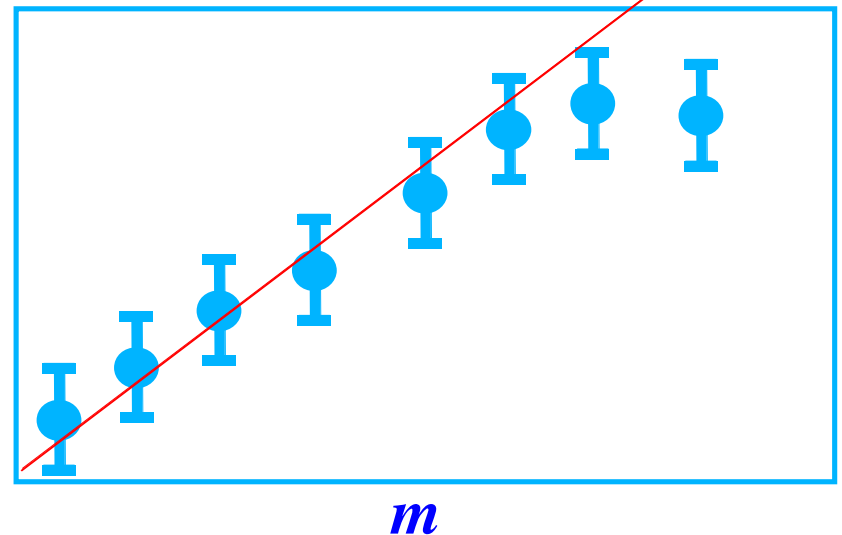
$$\frac{b^2}{4m^2} - \frac{k}{m} = 0$$

Construct and Test a Critical Damping System for a Spring

- Measure the spring constant k .
- Compute the damping coefficient b needed for critical damping.
- Adjust the air holes in the shock absorber to get the approximate b .
- Confirm $F = -bv$ assumption.
- Test spring plus shock absorber combination and optimize for critical damping.

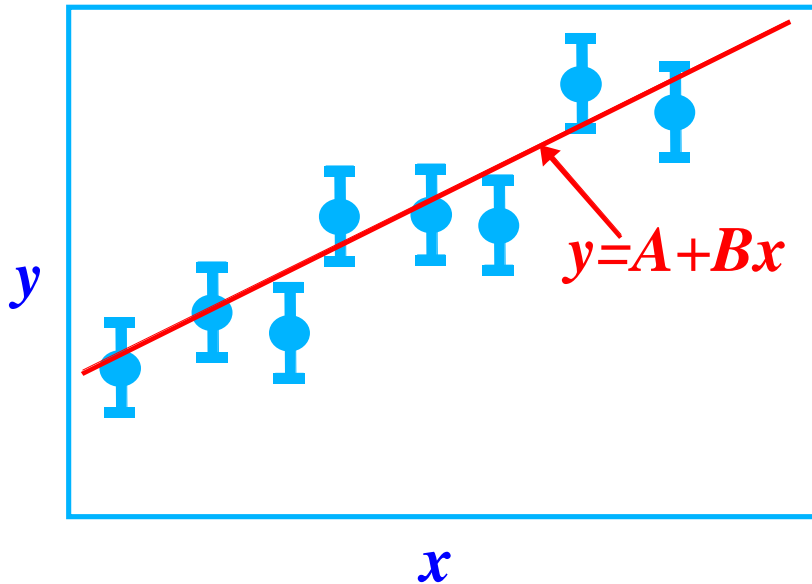


Terminal velocity is reached when $bv = mg$.



is damping linear in v ?

Least Squares Fitting



Best fit to a straight line as determined by the principle of maximum likelihood.

$$\chi^2 = \sum_{i=1}^n \frac{(y_i - [A + Bx_i])^2}{\sigma_y^2}$$

$$\frac{\partial \chi^2}{\partial A} = \frac{\partial \chi^2}{\partial B} = 0$$

Best fit parameters of line and their errors.

$$A = \frac{\sum x_i^2 \sum y_i - \sum x_i y_i \sum x_i}{\Delta}$$

$$\sigma_B^2 = \frac{n\sigma_y^2}{\Delta}$$

$$B = \frac{n \sum x_i y_i - \sum y_i \sum x_i}{\Delta}$$

$$\sigma_A^2 = \frac{\sum x_i^2 \sigma_y^2}{\Delta}$$

$$\Delta = n \sum x_i^2 - (\sum x_i)^2$$

Assume all points have same errors. If there are errors in both y and x , project errors onto y using the slope.

$$\sigma_y^2 \leftarrow \sigma_y^2 + B^2 \sigma_x^2$$

Deduce error by how well line fits data.

$$\sigma_y^2 = \frac{1}{n-2} \sum_{i=1}^n (y_i - A - Bx_i)^2$$

The ChiSquare Test

$$\chi^2 = \sum_{i=1}^n \frac{(y_i - f(x_i))^2}{\sigma_y^2}$$

Some Examples of $f(x)$ Number of Parameters

- **Weighted Average** **1**
- **Straight Line Fit** **2**
- **Parabola Fit** **3**
- **Exponential** **2**
- **Gaussian** **3**
- **Line with known slope** **1**

$$\langle \chi^2 \rangle = \mathbf{n}_{\text{d.o.f.}}$$

$$\mathbf{n}_{\text{d.o.f.}} = \mathbf{n}_{\text{data}} - \mathbf{n}_{\text{parameters}}$$

Define $\tilde{\chi}^2 = \frac{\chi^2}{\mathbf{n}_{\text{d.o.f.}}}$ and use this to test the goodness of fit of the data to the hypothesis of the fit function...

Experiment 4

In this experiment you will construct a device to measure the absolute value of a voltage through the measurement of a force. The actual measurements that you will make will be of mass, distance and time but the result will be a measurement of an electrical potential in Volts.

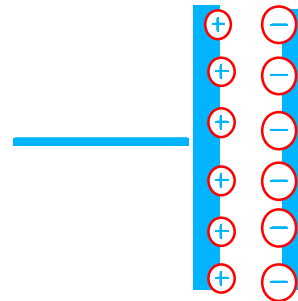
$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} \quad \text{two point charges}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{F}}{\text{m}}$$

fudge factor for unphysical units

$$V = \frac{\text{P.E.}}{Q} \quad \text{electric potential like potential energy}$$

Parallel Plate Capacitor



From Gauss's Law

$$E = \frac{Q}{A\epsilon_0}$$

Potential Difference

$$V = Ed = \frac{Qd}{A\epsilon_0}$$

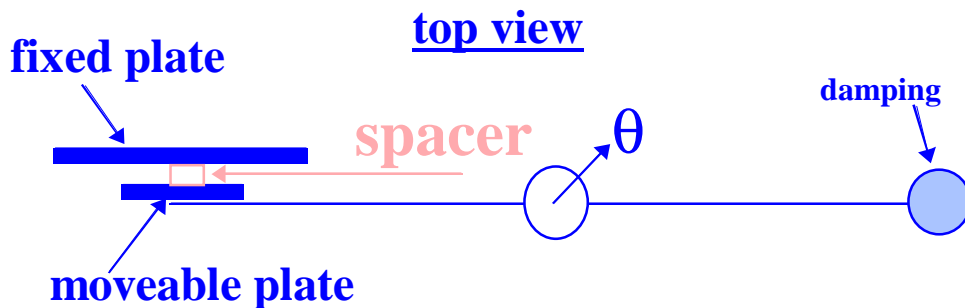
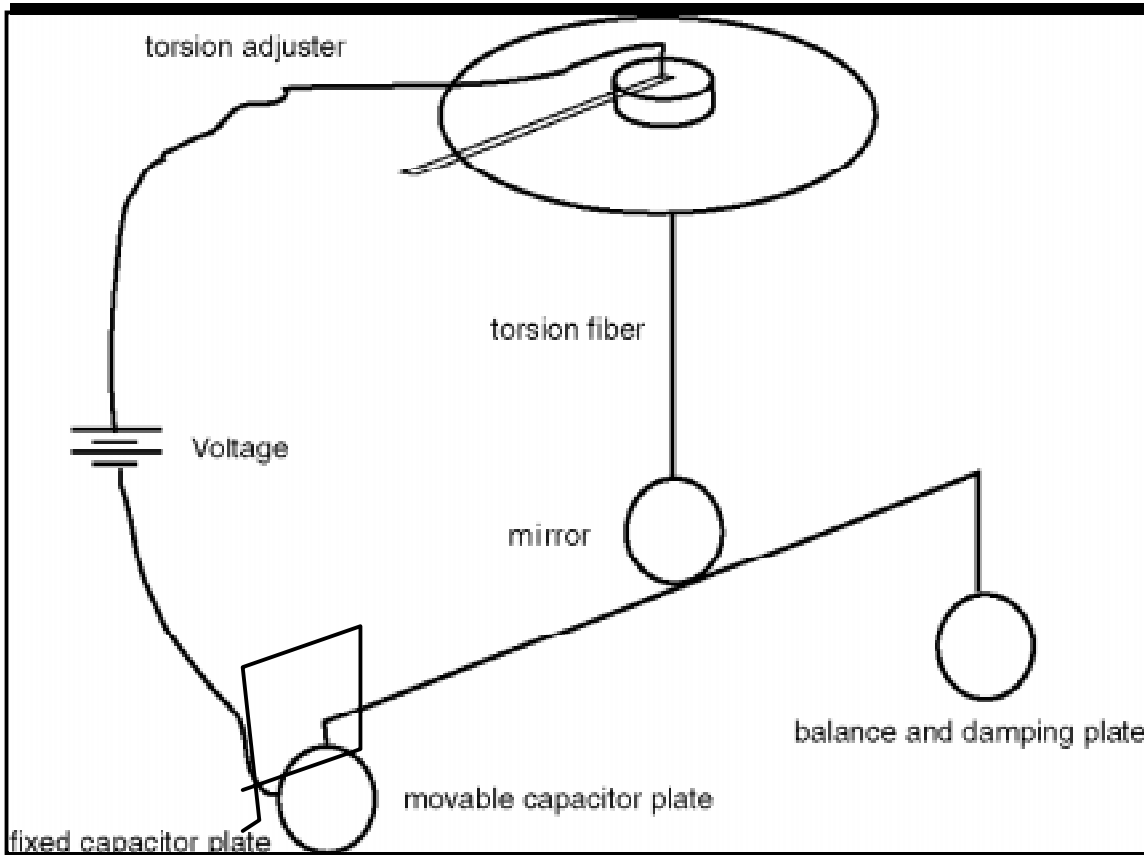
Force between plates

$$F = \frac{1}{2} \frac{A\epsilon_0}{d^2} V^2$$

Estimate force to be measured

$$F = \frac{1}{2} \frac{(A = 3\text{cm}^2)(8.8 \times 10^{-12}\text{F/m})}{(d = 0.1\text{cm})^2} (V = 1000\text{V})^2 = 1.2 \times 10^{-3}\text{N}$$

Calibrate a Voltmeter



- Set up the apparatus.
- Keep table dry.
- Strip wire well.
- Measure the spacer.
- Measure κ .
- Make the plates parallel for spacer in contact.
- Find Voltage that just causes plates to move apart.
- Try calibration at about 1000 Volts.
- Now get several measurements at lower voltage.
- Water must be stable.
- Move slowly.
- Protect your apparatus from air currents.
- Now use laser-mirror system to try to find stable equilibrium.
- How well could you check systematic differences in voltmeters this way?

Equilibrium Positions

$$F = -\frac{1}{2}A\epsilon_0 V^2 \frac{1}{d^2} \quad \text{electrostatic attraction}$$

$$F = \frac{\kappa\theta}{l} = \frac{\kappa}{l} \left(\theta_{\text{scale}} - \frac{d}{l} \right) \quad \text{force applied by torsion ballance}$$

spacer can ballance negative force when in contact. unstable

