Towards a Self-Consistent Theory of Turbulent Reconnection

Eun-jin Kim and P. H. Diamond

Department of Physics, University of California San Diego, La Jolla, CA 92093-0319

Abstract

Fast reconnection due to turbulent dissipation has long been hypothesized. The global reconnection rate is derived in the presence of 3D reduced magnetohydrodynamic turbulence taking into account the dynamic coupling between small and large–scale fields. The key result is that the reconnection rate remains inversely proportional to $R_m^{1/2}$ (R_m is the magnetic Reynolds number). In 2D limit, the global reconnection rate is shown to be enhanced over the Sweet–Parker result by a factor of magnetic Mach number. These results are consequences of mean square magnetic potential balance.

Typeset using REVTEX

Magnetic reconnection is thought to be a fundamental physical process underlying large scale magnetic energy release in space, astrophysical, and laboratory plasmas [1,2]. Symptomatic of a change in the global magnetic topology, magnetic reconnection plays an integral role in the dynamics of the magnetotail [3], the solar dynamo, solar coronal heating [4], and in the major disruption in tokamaks [5]. In view of the large Lundquist number $S \equiv v_A L/\eta$ in most astrophysical systems (i.e. $S \sim 10^{10}$ in the solar corona), the research on reconnection has focused on the construction of a *fast* reconnection model [6,7] (Here η and L are Ohmic diffusivity and the length scale of the current sheet). One such mechanism, turbulent reconnection has received much attention, since turbulent transport coefficients (which can be large for large S) are thought to act as effective dissipation coefficients, thus dramatically enhancing reconnection rates [5,8–10]. Indeed, the idea of an enhanced turbulent dissipation coefficient is now classic and has been widely invoked in flux diffusion and dynamo problems [4]. However, the recent studies indicated that the dynamical coupling between small and large scale fields (due to backreaction of small–scale magnetic fields on large–scale fields), which has been neglected in the conventional approach, can lead to a dramatic reduction in the *effective* turbulent transport coefficients. In particular, in 2D, flux diffusion, which is intimately related to the reconnection, was demonstrated to be reduced by [11,12]

$$\eta_{\rm eff} = \frac{\eta_k}{1 + R_m \langle B \rangle^2 / \langle u^2 \rangle}\,. \tag{1}$$

Here $\eta_{\rm eff}$ is the turbulent diffusivity; η_k is its kinematic value when the backreaction of small–scale magnetic fields is negligible; $\langle B \rangle$ is the large–scale magnetic field and $\langle u^2 \rangle$ the turbulent kinetic energy; $R_m = ul/\eta$ is the magnetic Reynolds number, with u and l being the characteristic amplitude and length scale of the velocity. The reduction in the flux diffusion is ultimately linked to the conservation of mean square magnetic potential in 2D ideal MHD, and the dynamic coupling between small and large scale fields is embedded in this conservation law.

In this letter, we present analytical results on the global reconnection rate in 3D Reduced MHD (RMHD), critically re-examining the naive idea of rapid reconnection due to turbulent dissipation. The motivation for considering 3D RMHD is to represent magnetic fields across current sheets that are not always antiparallel in real systems, and to avoid the null point problem inherent to the Sweet-Parker (called SP hereafter) slab model [13,14], thus rigorously justifying the assumption of incompressibility of the flow in the horizontal plane, perpendicular to a strong prescribed axial magnetic field $\mathbf{B}_0$. To gain an insights into this classic old problem in the simplest way, we adopt the following simple "SP + turbulence" model. First, we assume that turbulence is homogeneous and (almost) isotropic in the horizontal plane, by taking the reconnecting field in that plane to be weaker than fluctuating magnetic fields. By virtue of homogeneity and isotropy, the effect of turbulence can then be shown to appear principally in Ohm's law, among original SP balance relations [2]. In the case when the effect of turbulent diffusivity is larger than that of hyper-resistivity, which is usually the case in our model as will be seen later, the overall effect of turbulence effectively replaces the Ohmic diffusivity η by the total diffusivity $\eta_T = \eta + \eta_{\text{eff}}$. Therefore, the global reconnection rate in the presence of turbulence is roughly given by

$$v_r \sim \frac{v_A}{\sqrt{v_A L/\eta_T}} \,. \tag{2}$$

Note that when the backreaction is negligible, $\eta_{\text{eff}} \sim \eta_k \sim ul$, leading to fast reconnection rate v_r, independent of η. However, as the former becomes dynamically important, η_{eff} is no longer η_k, and its computation requires the self–consistent treatment of dynamical coupling between small and large scale fields. Our basic strategy is thus to compute the turbulent flux transport by accounting for this dynamical coupling and then use equation (2) to obtain the global reconnection rate.

The main equations governing 3D RMHD are as follows [15]:

$$\partial_t \psi + \mathbf{u} \cdot \nabla \psi = \eta \nabla^2 \psi + B_0 \partial_z \phi \,, \tag{3}$$

$$\partial_t \nabla^2 \phi + \mathbf{u} \cdot \nabla \nabla^2 \phi = \nu \nabla^2 \nabla^2 \phi + \mathbf{B} \cdot \nabla \nabla^2 \psi \,. \tag{4}$$

Here η and ν are Ohmic diffusivity and viscosity, respectively; $\mathbf{B} = B_0 \hat{z} + \mathbf{B}_H = B_0 \hat{z} + \nabla \times \psi \hat{z}$, in terms of a parallel component of the vector potential ψ; ϕ is a scalar potential of the (incompressible) velocity $\mathbf{u} = \nabla \times \phi \hat{z}$. Note that the conservation of the mean square potential is broken in 3D RMHD, albeit only linearly, due to the propagation of Alfvén waves along the axial magnetic field $B_0 \hat{z}$. Note also that the 3D RMHD ordering implies that $k_z/k_H \sim B_H/B_0 \sim \epsilon \ll 1$, where $\mathbf{k}_H = k_x \hat{x} + k_y \hat{y}$ and k_z are horizontal and vertical wavenumbers.

We decompose $\mathbf{B}_H$ into small and large scale components as $\mathbf{B}_H = \langle \mathbf{B}_H \rangle + \mathbf{b}$ (or, $\psi = \langle \psi \rangle + \psi'$), where $\langle \mathbf{B}_H \rangle$ is the reconnecting magnetic field (see Fig. 1), and assume $\langle \mathbf{u} \rangle = 0$. The statistics of $\mathbf{u}$ and $\mathbf{b}$ are assumed to be homogeneous and isotropic in x-y plane, homogeneous and reflectionally symmetric in the z direction, with no correlation between horizontal and vertical components. Here the assumption of isotropy is justified since the reconnecting field to be taken to be weak, i.e., $\langle B_H \rangle^2 \ll \langle u^2 \rangle$.

Turbulent dissipation coefficients are obtained by calculating flux $\Gamma_i = \langle u_i \psi' \rangle = -\eta_{\text{eff}}\ \partial_i \langle \psi \rangle + D_H \partial_i \nabla^2 \langle \psi \rangle$, which appears in the mean field equation:

$$\partial_t \langle \psi \rangle = \eta_T \nabla^2 \langle \psi \rangle - D_H \nabla^2 \nabla^2 \langle \psi \rangle \,. \tag{5}$$

Here $\eta_T = \eta + \eta_{\text{eff}}$ is the total diffusivity, η_{eff} turbulent diffusivity, and D_H hyper-resistivity. In order to evaluate this flux, we employ two models — quasi–linear closure with τ approximation [12] and eddy–damped fluid model based on large viscosity [16]. Since the calculation is lengthy, we sketch only a few main steps and provide the final results here, leaving the details for the extended version of this paper.

We first consider a quasi–linear closure with τ approximation. The basic idea of this model is to compute Γ_i as $\Gamma_i = \epsilon_{ij3} \langle \partial_j \phi \psi' \rangle = \epsilon_{ij3} \langle \partial_j \phi \delta \psi' - \delta \phi \partial_j \psi' \rangle$, where unity magnetic Prandtl number is assumed for the equal splitting between $\langle \partial_j \phi \delta \psi' \rangle$ and $\langle \delta \phi \partial_j \psi' \rangle$. $\delta \phi$ and $\delta \psi'$ are calculated from the equations for ϕ and ψ', by introducing the (same) correlation time τ for ϕ and ψ' [12]. Then, upon using the aforementioned statistical property of $\mathbf{u}$ and $\mathbf{b}$, we obtain the flux in the following form:

$$\Gamma_i = -\frac{\tau}{2} \left[(\langle u^2 \rangle - \langle b^2 \rangle) \partial_i \langle \psi \rangle - \langle \psi'^2 \rangle \partial_i \nabla^2 \langle \psi \rangle \right] \,. \tag{6}$$

The terms on the right hand side of equation (6) represent, from the left, kinematic turbulent diffusion by fluid advection of the flux, the flux coalescence due to the backreaction of small–scale magnetic fields with the (negative) diffusion coefficient proportional to $\langle b^2\rangle$, and the hyper-resistivity due to the gradient of a large–scale current $\langle J\rangle = -\nabla^2\langle\psi\rangle$ ($J\hat{z} = \nabla\times\mathbf{B}_H$) [17]. Note that the negative diffusivity and hyper-resistivity together conserve total $\langle\psi'^2\rangle$, while shuffling the $\langle\psi'^2\rangle$ spectrum toward large scales.

The above equation for the flux is represented in terms of (small–scale) magnetic energy $\langle b^2\rangle$, which is unknown. Thus, the determination of the flux requires additional relation between the flux and $\langle b^2\rangle$. The key point here is that the latter can be obtained by invoking a statistically stationary state of small–scale fields. That is, when $\partial_t\langle\psi'^2\rangle = 0$, the equation for $\langle\psi'^2\rangle$ is reduced to

$$\langle(\partial_i\psi')^2\rangle = \langle b^2\rangle = \frac{1}{\eta}\left[-\Gamma_i\partial_i\langle\psi\rangle + B_0\langle\psi'\partial_z\phi\rangle\right] , \tag{7}$$

by neglecting the boundary terms [18]. The use of a quasi-linear closure for the last term in equation (7) as $\langle\psi'\partial_z\phi\rangle = \langle\delta\psi'\partial_z\phi - \partial_z\psi'\delta\phi\rangle$ then gives us

$$\langle b^2\rangle = \frac{1}{\eta}\left[-\Gamma_i\partial_i\langle\psi\rangle + \tau\xi_v B_0^2\langle u^2\rangle\right] / \left(1 + \frac{\tau\xi_b}{\eta}B_0^2\right) . \tag{8}$$

Here $\xi_v \equiv L_{vH}^2/L_{vz}^2$ and $\xi_b \equiv L_{bH}^2/L_{bz}^2$; L_{vH} and L_{vz} are the characteristic horizontal and vertical scales of $\mathbf{u}$, and L_{bH} and L_{bz} are those of $\mathbf{b}$. Thus, the flux Γ_i simply follows from equations (6) and (8) as

$$\Gamma_i = -\eta_k \frac{\left[1 + \frac{\tau}{\eta}B_0^2(\xi_b - \xi_v)\right]\partial_i\langle\psi\rangle - \frac{\tau L_{bH}^2}{\eta}\xi_v B_0^2\partial_i\nabla^2\langle\psi\rangle}{1 + \frac{\tau}{\eta}\left[\frac{1}{2}\langle B_H\rangle^2 + \xi_b B_0^2 - \frac{L_{bH}^2}{2}\langle J\rangle^2\right]} , \tag{9}$$

where $\eta_k = \tau\langle u^2\rangle/2$ is the kinematic turbulent diffusivity. Note the last term in the numerator and denominator in equation (9) comes from the hyper-resistivity [17]. Equation (9) is the flux transport in 3D RMHD, which generalizes the 2D MHD result [11,12]. In the limit as $\mathbf{B}_0 \to 0$ and $\langle\mathbf{B}_H\rangle \to 0$, the flux reduces to the kinematic value $\Gamma_i = -\eta_k\partial_i\langle\psi\rangle$. The full 2D MHD result can be obtained by taking the limit $\mathbf{B}_0 \to 0$, which will reproduce equation (1) [11,12].

We now examine the various terms in equation (9) to estimate v_r in 3D RMHD. First, note that $\xi_v - \xi_b$ in equation (9) can be taken to be zero (i.e., $\xi_v \sim \xi_b$), since the scales of $\mathbf{b}$ and $\mathbf{u}$ are likely to be comparable because of the assumption of unity magnetic Prandtl number made here. We also note $\xi_b B_0^2 \sim \langle B_H^2 \rangle$, due to 3D RMHD ordering ($k_z/k_H \sim B_H/B_0 \sim \epsilon < 1$), and $\xi_b B_0^2 \sim \langle b^2 \rangle \gg \langle B_H \rangle^2$. Thus, the ratio of the last term in the numerator (hyper–resistivity term) to the first term (turbulent diffusivity term) is estimated to be $\tau \langle b^2 \rangle L_{bH}^2/(\eta L_{BH}^2) \sim (L_{bH}/L_{BH})^2 R_m$, where L_{BH} is the characteristic horizontal length scale of $\langle \mathbf{B}_H \rangle$, and $\langle b^2 \rangle \sim \langle u^2 \rangle$ is used. If $L_{bH}/L_{BH} \sim R_m^{-\chi}$, $\chi \sim 1/2$ in 2D and $\chi \gtrsim 1/2$ in 3D as suggested by [19]. Thus, $(L_{bH}/L_{BH})^2 R_m$ is likely to be at most of order unity. For this reason, the hyper-resistivity term will be neglected in the following analysis. The dominant term in the square brackets in the denominator of equation (9) is $\xi_b B_0^2 \sim \langle b^2 \rangle$ as $L_{bH}^2 \langle J \rangle^2 \sim (L_{bH}/L_{BH})^2 \langle B_H \rangle^2 < \langle B_H \rangle^2$. Alternatively, the effect of B_0 seems to be stronger than that of $\langle \mathbf{B}_H \rangle$ in 3D RMHD, resulting in a more severe reduction in the flux than in 2D. This is probably due to radiative losses of Alfvén waves along the axial magnetic field in 3D RMHD reducing the mixing in the horizontal plane, thereby lowering $\eta_{\rm eff}$.

Now, upon using the estimates mentioned above, we obtain

$$\eta_{\rm eff} \sim \eta_k \frac{1}{1 + \tau \langle b^2 \rangle / \eta} \sim \eta_k \frac{1}{1 + 2 R_m \langle b^2 \rangle / \langle u^2 \rangle} \sim \frac{\eta}{2}, \tag{10}$$

where we used $R_m = \eta_k/\eta$ and the following estimate on $\langle b^2 \rangle$

$$\langle b^2 \rangle \sim \langle u^2 \rangle - \frac{\eta}{\tau} \sim \langle u^2 \rangle \left[1 - \frac{1}{2R_m} \right]. \tag{11}$$

Therefore, to leading order, turbulent diffusivity is just that given by Ohmic diffusivity! The reconnection rate is now found by neglecting hyper-resistivity and by inserting equation (10) into (2) with $\eta_T = \eta + \eta_{\rm eff}$

$$v_r \sim \frac{v_A}{\sqrt{v_A L/\eta}}. \tag{12}$$

That is, the reconnection rate has the original SP scaling with η.

It is of interest to contrast this result to the 2D case, by taking $B_0 = 0$ in equation (9). In that case, a similar order of estimate leads to the global reconnection rate

$$v_r \sim \frac{v_A}{\sqrt{v_A L/\eta}} \frac{u}{v_A}\,, \tag{13}$$

which is larger than SP by a factor of magnetic Mach number $M_A = u/v_A$.

We now show that the η scaling of the reconnection rate (12) is rather robust feature of 3D RMHD, by considering our second model, that of an eddy-damped fluid [16]. This is the simplest model within which the nonlinear effect of the back–reaction can rigorously be treated, without having to assume the presence of fully developed MHD turbulence, to invoke momentum closure, or to introduce an arbitrary correlation time for the fluctuating fields. Despite its limited applicability to a system with large viscosity, it could still be quite relevant to small scale fields in the Galaxy where $\nu \gg \eta$.

The key idea of this model lies in the neglect of the nonlinear advection terms in the momentum equation due to small $R_e = ul/\nu$, which permits to split the velocity $\mathbf{u}$ into two components; the first — random velocity $\mathbf{v}$ — is solely governed by the random forcing, and the second — induced velocity $\mathbf{v}'$ — is governed by the Lorentz force only. The $\mathbf{v}$ is assumed to be delta correlated in time with the correlation time τ_0, in addition to satisfying the spatial homogeneity and isotropy in the x-y plane. The induced velocity ($\mathbf{v}' = \nabla \times \phi_I \hat{z}$) is obtained by solving $\nu \nabla^2 \phi_I + \mathbf{B} \cdot \nabla \nabla^2 \psi = 0$ for ϕ_I in Fourier space. Some of crucial steps, leading to the flux, are as follows. First, the equation for $\langle \psi \rangle$ and $\langle \psi^2 \rangle$ can be obtained by inserting both velocity $\mathbf{v}$ and $\mathbf{v}'$ in the equation (3) and by taking averages. The mean field equation for $\langle \psi \rangle$ determines the flux Γ_i which again involves $\langle b^2 \rangle$. Then, upon the imposition of the stationarity condition on $\langle \psi'^2 \rangle$, similar algebra performed in the quasi-linear closure model gives

$$\Gamma_i = -\tilde{\eta}_k \frac{\left[1 + \frac{\kappa}{\eta\nu} B_0^2 (\xi_b - \xi_v)\right] \partial_i \langle \psi \rangle - \frac{2\kappa\gamma}{\eta\nu} \xi_v B_0^2 \partial_i \nabla^2 \langle \psi \rangle}{1 + \frac{\kappa}{\eta\nu} \left[\xi_b B_0^2 + \frac{1}{2} \langle B_H \rangle^2 - \gamma \langle J \rangle^2\right]}\,, \tag{14}$$

where $\tilde{\eta}_k = \tau_0 \langle v^2 \rangle / 4$ is the kinematic diffusivity due to the random velocity $\mathbf{v}$, $\kappa \equiv L_{bH}^2$, and $\gamma \equiv L_{bH}^4 / L_{bz}^2$. Note that L_{vH} and L_{vz} ($\xi_v = L_{vH}^2 / L_{vz}^2$) are the scales associated with $\mathbf{v}$. In the limit $B_0 \to 0$ and $B_H \to 0$, equation (14) recovers the 2D hydrodynamic result with the kinematic diffusivity $\tilde{\eta}_k = \tau_0 \langle v^2 \rangle / 4$. The limit $B_0 \to 0$ alone leads to 2D MHD case.

To estimate the flux in the general case, we assume $\xi_v = \xi_b$ in the following. As discussed earlier, the dominant term in the square brackets in the denominator of equation (14) is $\xi_b B_0^2 \sim \langle b^2 \rangle$. The last term in the numerator and denominator, due to the hyper-resistivity, now comes with a multiplicative factor $\gamma = \kappa \xi_b$ with $\xi_b = L_{bH}^2 / L_{bz}^2 \ll 1$. Thus, the effect of hyper-resistivity can be neglected as compared to other terms in equation (14). Note that since $L_{bz} \to \infty$ in 2D limit, $\gamma \to 0$ in 2D MHD. That is, there is no contribution from the hyper-resistivity to the flux in 2D in this model. The above estimates then give us

$$\eta_{\text{eff}} \sim \tilde{\eta}_k \frac{1}{1 + \frac{\kappa}{\nu\eta}\langle b^2 \rangle} \sim \frac{\eta}{2}, \tag{15}$$

where the estimate on $\langle b^2 \rangle \sim \eta\nu(2R_m - 1)/\kappa$ was used. Therefore, the reconnection rate is again given by equation (12) [20]! In comparison, the global reconnection rate in the 2D limit can be shown to be

$$v_r \sim \frac{1}{\sqrt{R_e}} \frac{v_A}{\sqrt{v_A L/\eta}} \frac{u}{v_A}. \tag{16}$$

The final point we would like to address is the incompatibility of turbulent fluid–magnetic energy equipartition and stationarity of mean square magnetic potential. Note that equation (11), naturally following from the stationarity of $\langle \psi'^2 \rangle$, does not satisfy the exact equipartition $\langle b^2 \rangle = \langle u^2 \rangle$, which is often assumed in Alfvénic turbulence [21]. Moreover, it can easily be shown by using a quasi–linear model that the assumption of Alfvénic turbulence are not compatible with stationarity of $\langle \psi'^2 \rangle$. Therefore, in general, stationarity of $\langle \psi'^2 \rangle$ and exact Alfvénic equipartition cannot be simultaneously realized.

To summarize, we have self-consistently computed turbulent diffusivity η_{eff} and hyper-resistivity D_H in 3D RMHD, by exploiting the 'linearly-broken' conservation of $\langle \psi^2 \rangle$ and by invoking the stationarity of $\langle \psi'^2 \rangle$. We obtained $\eta_{\text{eff}} \sim \eta_k/(1 + 2R_m\langle b^2 \rangle/\langle u^2 \rangle) \sim \eta/2$ in the quasi-linear closure model and $\eta_{\text{eff}} \sim \tilde{\eta}_k/(1 + R_m R_e \langle b^2 \rangle/\langle v^2 \rangle) \sim \eta/2$ in the eddy-damped fluid model; the effect of D_H was demonstrated to be at most comparable to that of η_{eff}. Then, SP type balance relations led to the global reconnection rate $v_r \sim v_A\sqrt{v_A L/\eta}$ in both models, which is the same as in SP. In the 2D limit, v_r was shown to be enhanced over the

SP result roughly by a factor of magnetic Mach number. This is probably due to radiative losses of Alfvén waves along the axial magnetic field in 3D RMHD reducing $\eta_{\rm eff}$ and v_r.

We note that in full 3D MHD, $\langle\psi^2\rangle$ conservation is broken nonlinearly unlike in 3D RMHD. Therefore, the global reconnection rate in 3D MHD is likely to be different from that in 3D RMHD. Moreover, non-stationarity of small-scale fields, as indicated by bursty 'plasmoid ejection' events observed in [9], may alter the conservation law constraints on reconnection by modifying the relation between $\langle\psi'^2\rangle$ (or $\langle b^2\rangle$) and Γ_i. A key question is whether the behavior of small–scale magnetic fields is periodic (as in a limit cycle) or temporally chaotic and intermittent. Some analytical progress may be made in the quasi–linear or cyclic case, which will be studied in a future paper. We conclude by noting that ultimately, a numerical simulation should be performed to investigate our predictions of the flux diffusion and global reconnection rate in 3D RMHD.

Acknowledgments We thank E. Zweibel for bringing this problem to our attention and E. Zweibel and A.S. Ware for stimulating discussions. This research was supported by U.S. DOE FG03-88ER 53275. P.H. Diamond also acknowledges partial support from the National Science Foundation under Grant No. PHY99-07949 to the Institute for Theoretical Physics at U.C.S.B., where part of this work was performed. E. Kim acknowledges partial support from HAO/NCAR where part of this work was completed.

REFERENCES

[1] V. Vasyliunas, Rev. Geophys. Space Phys. **13**, 303 (1975).

[2] D. Biskamp, Nonlinear Magnetohydrodynamics (Cambridge: Cambridge Univ. Press) (1993).

[3] T. Forbes and E. Priest, J. Geophys. Res. **88**, 863 (1983).

[4] E.N. Parker, Cosmical magnetic fields (Oxford: Oxford University Press) (1979).

[5] P.H. Diamond, R.D. Hazeltine, Z.G. An, B.A. Carreras, and H.R. Hickes, Phys. Fluids **27**, 1449 (1984).

[6] H.E. Petschek, Physics of Solar Flares, AAS-NASA symposium (NASA SP-5), ed. W.N. Weiss (Greenbelt, MD:NASA), 425 (1964); D. Biskamp, Phys. Fluids **29**, 1520 (1986); R.M. Kulsrud, Astro-ph/0007075 (2000).

[7] A.Y. Aydemir, Phys. Fluids B **4**, 3469 (1992); X. Wang and A. Bhattacharjee, Phys. Rev. Lett. **70**, 1627 (1993); M.A. Shay, J.F. Drake, B.N. Rogers, and R.E. Denton, Geophys. Res. Lett. **26**, 2163 (1999).

[8] H.R. Strauss, Astrophys. J. **326**, 412 (1988).

[9] W.H. Matthaeus and S.L Lamkin, Phys. Fluids **29**, 2513 (1986).

[10] A. Lazarian and E.T. Vishniac, Astrophys. J. **517**, 700 (1999).

[11] F. Cattaneo and S.I. Vainshtein, Astrophys. J. **L21**, 376 (1991).

[12] A.V. Gruzinov and P.H. Diamond, Phys. Rev. Lett. **72**, 1651 (1994).

[13] E.N. Parker, J. Geophys. Res. **62**, 509 (1957).

[14] P.A. Sweet, P.A. in IAU Symp. 6, Electromagnetic Phenomena in Cosmical Plasmas, ed. B. Lehnert(New York: Cambridge Univ. Press), 123 (1958).

[15] H.R. Strauss, Phys. Fluids **19**, 134 (1976).

[16] E. Kim, Phys. Lett. A **259**, 232 (1999).

[17] A. Boozer, J. Plasma Phys. **35**, 133 (1986); A. Bhattacharjee and Y. Yuan, Astrophys. J. **449**, 739 (1995).

[18] For the effect of boundary terms and non–stationarity, see E.G. Blackman and G.B. Field, Astrophys. J. **534**, 984 (2000).

[19] S.I. Vainshtein and F. Cattaneo, Astrophys. J. **393**, 165 (1992).

[20] Note that a multiplicative correction factor to the reconnection rate due to high viscosity [2] is neglected here since its dependence on ν is weak with 1/4 power.

[21] P. Goldreich and S. Sridhar, Astrophys. J, **485**, 680 (1997) and references therein.

FIGURES

FIG. 1. Configuration in 3D RMHD. $\mathbf{B}_0$ is a strong axial magnetic field pointing in the z direction, and $\pm\mathbf{B}_H$ are reconnecting (large–scale) magnetic fields in the x-y plane. Panel (a) shows the projection in the x-y plane, and Panel (b) in the y-z plane.