Transport Reduction by Shear flows in Dynamical Models

Eun-jin Kim[1], P. H. Diamond[1], and T.S. Hahm[2]

[1] *Department of Physics, University of California, San Diego, La Jolla, CA 92093-0319*

[2] *Princeton Plasma Laboratory, Princeton University, P.O. Box 451, Princeton, NJ 08543*

Abstract

We study the reduction in the transport of particles and heat by a strong mean shear flow, in the context of interchange and ion-temperature gradient turbulence models. Compared to passive scalar transport, a stronger reduction in the transport (scaling with the shearing rate Ω as $\propto \Omega^{-3} \ln \Omega$) results from a severe reduction in the amplitude of turbulent velocity in both models. However, the cross-phase is only modestly reduced, as in the scalar field case. These results are in qualitative agreement with the results from both gyrokinetic and gyrofluid simulations of toroidal ITG turbulence, but contradict recent claims in some literature and highlight the importance of the detailed properties of the flow in determining the overall transport level.

Typeset using REVTEX

I. INTRODUCTION

The suppression of anomalous transport by shear flows plays an integral role in the formation of transport barriers (such as the L-H transition and internal transport barriers) in magnetically confined plasmas.[1] In particular, predictive modeling of L-H transition requires a quantitative information as to exactly how much of the transport flux of interest is reduced, for a given shear strength. Recent experimental measurements, prior to and after the L-H transition, seem to suggest a strong suppression of particle or heat transport, with scaling $\propto \Omega^{-\alpha}$ where $1.6 \lesssim \alpha \lesssim 4.8$.[2,3] Despite these experimental measurements, concrete theoretical work on the prediction of such scalings, beyond the simple passive scalar model,[4,5] is still lacking. Such a theoretical work will be extremely useful to understand basic physical mechanism leading to a particular scaling, by disentangling the complexity of different physical effects in a real system, thereby identifying key effects that are most pertinent to transport suppression. For instance, whether the reduction in the transport is mainly due to the reduction in turbulence amplitude, or due to the reduction in the cross-phase, in principle, can, and should, be predicted.

One of the most intriguing questions regarding shear suppression is whether there is a universal scaling of transport with shear strength (Ω) in the limit of strong shear, which is *independent* of the *details* of turbulence. If it were the case, this universal scaling could easily be obtained through the study of any turbulence model, the simplest of which is, obviously, passive scalar fields that are advected by a given random turbulent flow and a shear flow. However, our previous study[4] indicated that the predicted suppression of scalar field transport ($\propto \Omega^{-1}$) is too weak to be consistent with experimental measurements. This is not surprising in view of the limitation of the passive scalar model within which the random turbulent flow is arbitrary prescribed. In addition, it was pointed out that the exact scaling depends on the properties of the random turbulent flows, such as the magnitude of their correlation time (τ_c) relative to the shearing time ($\tau_\Omega = \Omega^{-1}$).[4] The origin of this dependence can be traced back to identifying the dominant source of irreversibility, which is necessary for a non-trivial flux. Furthermore, the amplitude of the velocity is also arbitrarily fixed in the passive scalar model. In short, the dynamics of the flow is one of the missing ingredients in the passive scalar model, which however can be critical to determining transport levels.

The purpose of the paper is to study transport reduction in realistic situations where a turbulent flow (electric potential gradient) evolves dynamically. As simple dynamical models, we shall consider interchange and simple ion temperature gradient (ITG) turbulence models to study the transport of particle and heat, respectively. The analysis shall be limited to the strong shear case where the shearing rate exceeds the nonlinear decorrelation rate of turbulence. This will justify quasi-linear analysis that is used. In order to simplify the analysis, a free energy source in both models will be treated as a part of random noise. As a consequence, the saturation level of turbulence amplitude is essentially set by the noise and dissipation.

The principal conclusions of this paper are as follows (see Table 1):

i) A strong reduction in the transport of particles (from interchange turbulence) and heat (from ITG turbulence) results from a severe reduction in the amplitude of velocity in both models. The very strong transport scaling with respect to the shearing rate ($\propto \Omega^{-3}\ln\Omega$) offers an explanation of some experimental results that the ion heat transport gets reduced to the level of neoclassical value due to the $\mathbf{E}\times\mathbf{B}$ shear[1] without a need to invoke an ad-hoc, but popular turbulence quenching rule involving the linear growth rate.

ii) On the other hand, the reduction in cross-phase is very weak ($\propto \Omega^{-1/6}\ln\Omega$). This prediction is much closer to the recent results from ITG simulations[6] where no relevant variation of the cross-phase is observed, than to the results from a previous simulation[7] and the claims of a very strong reduction in Ref. 5.

iii) The generation of mean flow through Reynolds stress (i.e., inverse cascade), and how this Reynolds stress driving itself is reduced by shear as its amplitude becomes large.

The remainder of the paper is organized as follows. Section II presents the particle transport in interchange turbulence model. The reduction in heat transport by shear flows in ITG is discussed in Sec. III. Section IV contains our conclusion and discussions.

II. PARTICLE TRANSPORT IN INTERCHANGE TURBULENCE MODEL

To study particle transport in the strong shear limit, we assume cold ions and consider the quasi-linear evolution of flute-like perturbations of particle density n and vorticity $\omega = \nabla\times\mathbf{v}$ in a 2D plane, which are subject to a given (poloidal) shear flow $U_0(x)\hat{y}$ ($U_0 = -x\Omega$) and effective gravity $\mathbf{g} = g\hat{x}$ (due to magnetic curvature, etc):

$$\partial_t n + U_0\partial_y n = -v_x\partial_x N_0 + D\nabla^2 n + f\,, \tag{1}$$

$$\partial_t\omega + U_0\partial_y\omega = -g\partial_y n/N_0 + \nu\nabla^2\omega\,. \tag{2}$$

Here, $\mathbf{u} = \mathbf{v} + U_0(x)\hat{y}$ is the total velocity with $\mathbf{v} = -(c/B_0)\nabla\phi\times\hat{z}$, and $N = N_0(x) + n$ is the total density where $N_0(x)$ are n are the mean background density and fluctuation; x and y represent the local radial and poloidal directions, respectively, perpendicular to a magnetic field $\mathbf{B} = B_0\hat{z}$; D and ν capture the coherent nonlinear interaction (i.e., "eddy diffusivity and viscosity") as well as molecular dissipation while f represents a noise due to incoherent nonlinear interaction and external particle source. Within this model, density is simply advected by a turbulent flow ($\mathbf{v}$) and a given shear flow $U_0(x)\hat{y}$, similar to passive scalar field, while the flow ($\mathbf{v}$) is dynamically determined. As is well-known, in the absence of shear flow and dissipation, this system is linearly unstable when the effective gravity acts against the background density gradient (i.e. $g(\partial_x N_0) < 0$), with a linear growth rate $\gamma_g = \sqrt{(-\partial_x N_0/N_0)g}$.

Before proceeding to see how the shear flow reduces transport and turbulence amplitude, we shall first examine how it alters the linear instability. We incorporate the main shearing effect non-perturbatively by following a particle trajectory along which radial wavenumber k_x linearly increases in time (i.e., $k_x(t) = k_x(0) + k_y t\Omega$). For simplicity, this can be achieved by using the Gabor[8] and Fourier Transforms (GFT) in the x and y directions, respectively: $GFT[\psi(\mathbf{x},t)] = \hat{\psi}(\mathbf{k},x,t) = \int dx'\, f(|x-x'|)e^{ik_x(x-x')}\int dy'\, e^{ik_y y'}\psi(\mathbf{x}',t)$. Here, $f(x)$ is a function at scale λ, filtering out the information on scales larger than λ, with λ lying between characteristic scales for fluctuations and mean flows. Using the Gabor transform in the radial

(x) direction only is sufficient to capture shearing effect that depends on x ($U_0 = -x\Omega$) and also to describe turbulence, which is radially localized around resonant surfaces (due to magnetic shear). Under this GFT, Eqs. (1) and (2) are rewritten as:

$$(D_t + ik_y U_0)\hat{n} = -\hat{v}_x \partial_x N_0 - D(k_x^2 + k_y^2)\hat{n} + \hat{f}\,, \tag{3}$$

$$(D_t + ik_y U_0)\hat{\omega} = -(g/N_0) ik_y \hat{n} - \nu(k_x^2 + k_y^2)\hat{\omega}\,, \tag{4}$$

where $D_t = \partial_t + k_y \Omega \partial_{k_x}$ is the total time derivative, and $\Omega = -\partial_x U_0$ is assumed to be positive, without loss of generality. Note that $D_t k_x = k_y \Omega$, by the eikonal equations. In the case $D = \nu = f = 0$, the coupled equations can be easily solved in the long time limit (i.e., for large $R = k_x/k_y = \Omega t$), to obtain the solution $\hat{\omega} \sim R^\alpha$, where

$$\alpha = \frac{1}{2}\left[1 \pm (1 + 4\gamma_g^2/\Omega^2)^{1/2}\right]\,. \tag{5}$$

Note that the power-law in $R = k_x/k_y$, instead of exponential, for $\hat{\omega}$ is due to the wind-up by the shear flow. In the strong shear limit ($\gamma_g/\Omega \ll 1$), $\alpha \sim 1 + (\gamma_g/\Omega)^2$, $-(\gamma_g/\Omega)^2$. Thus, shearing softens the exponential behavior (of both linearly unstable and stable modes) to linear behavior, on account of the eikonal phase wind-up induced by the shear flow. Note that these modes may not be eigenmodes because of the presence of a shear flow,[9] and also that shearing does not stabilize the unstable mode completely. However, if growth to nonlinearity occurs rapidly enough for non-eigenmode perturbations, the details of their origin are irrelevant.

To obtain the scaling of various correlation functions ($\langle n v_x \rangle$, $\langle n^2 \rangle$, $\langle v_x^2 \rangle$, and $\langle v_x v_y \rangle$) with shearing rate, we further simplify the analysis by assuming $D = \nu$ and by treating the source of free energy $v_x \partial_x N_0$ in Eq. (1) as a part of the noise f. Under these assumptions, the solutions for $\hat{n}$ and $\hat{\omega}$ are immediately obtained as:

$$\hat{n}(\mathbf{k}, x, t) = \int_{-\infty}^{t} dt_1 d^2k_1 \hat{g}(\mathbf{k}, t; \mathbf{k}_1, t_1) \hat{f}(\mathbf{k}_1, x, t_1)\,, \tag{6}$$

$$\hat{v}_x(\mathbf{k}, x, t) = (\frac{g}{N_0}) \int_{-\infty}^{t} dt_1 d^2k_1 (t - t_1) \frac{k_y^2}{k^2} \hat{g}(\mathbf{k}, t; \mathbf{k}_1, t_1) \hat{f}(\mathbf{k}_1, x, t_1)\,, \tag{7}$$

with the Green's function $\hat{g}$ given by

$$\hat{g}(\mathbf{k},t:k_1,t_1) = \delta(k_x - k_{1x} - k_y\Omega(t-t_1))\delta(k_y - k_{1y})\exp\{-iU_0k_{1y}(t-t_1)\}$$
$$\times \exp\left\{-D\left(k_y^2 t + \frac{k_x^3}{3\Omega k_y}\right)\right\}\exp\left\{D\left(k_{1y}^2 t_1 + \frac{k_{1x}^3}{3\Omega k_{1y}}\right)\right\}. \quad (8)$$

Various correlation functions now simply follow from Eqs. (6), (7), and (8) for a noise with a given statistics:

$$\langle \hat{f}(\mathbf{k}_1,x,t_1)\hat{f}(\mathbf{k}_2,x,t_2)\rangle \sim (2\pi)^2\delta(\mathbf{k}_1+\mathbf{k}_2)\tilde{\psi}(\mathbf{k}_2,t_2-t_1)\,, \quad (9)$$

where $\tilde{\psi}$ is the Fourier transform of $\psi(\mathbf{r},t) \equiv \langle f(\mathbf{x},t_1)f(\mathbf{x}+\mathbf{r},t_2)\rangle$.

Since the noise originates from the incoherent nonlinear interaction and free energy source as well as external sources, it is reasonable to assume that its correlation time τ_f is shorter than the nonlinear decorrelation time $\tau_D = 1/Dk^2$. And since we are interested in the strong shear limit such that $\tau_\Omega \ll \tau_D$, depending on the ordering between τ_f and τ_Ω, we have the following two possibilities: (i) $\tau_f < \tau_\Omega \ll \tau_D$ and (ii) $\tau_\Omega < \tau_f \ll \tau_D$. The first case (i) corresponds to a noise with a short (delta) correlation time, where the irreversibility, leading to non-trivial transport is largely due to the noise randomness. In this case, the effect of shear is minimal as the noise field changes before the shearing can act. In comparison, in the second case (ii), the overlap of resonant layers is the main source of the irreversibility, and coherent shearing over time $t < \tau_f$ gives a stronger effect of the shear flow. In the following, the various correlation functions shall be presented in these two cases in the long time limit (i.e., as $t \to \infty$) when $f(\mathbf{k})$ is dominated by modes with $k_x \ll k_y$.

In the first case (i) where $\tau_f < \tau_\Omega$, we approximate the noise correlation function $\tilde{\psi}(\mathbf{k},t_1-t_2) = \psi(\mathbf{k})\delta(t_1-t_2)$. Then, straightforward algebra gives us the following results to leading order in $\xi \equiv Dk_1^2/\Omega \ll 1$:

$$\langle nv_x\rangle \sim \frac{1}{(2\pi)^2\Omega^2}\frac{g}{N_0}\int d^2k_1\psi(\mathbf{k}_1)\ln\xi^{-1/3} \propto \Omega^{-2}\ln\Omega\,, \quad (10)$$

$$\langle n^2\rangle \sim \frac{1}{(2\pi)^2\Omega}\int d^2k_1\psi(\mathbf{k}_1)\frac{1}{3}\Gamma\left(\frac{1}{3}\right)\left(\frac{3}{2\xi}\right)^{1/3} \propto \Omega^{-2/3}\,, \quad (11)$$

$$\langle v_x^2\rangle \sim \frac{1}{(2\pi)^2\Omega^3}\left(\frac{g}{N_0}\right)^2\int d^2k_1\psi(\mathbf{k}_1)\frac{\pi}{4} \propto \Omega^{-3}\,, \quad (12)$$

$$\langle v_xv_y\rangle \sim -\frac{1}{(2\pi)^2\Omega^3}\left(\frac{g}{N_0}\right)^2\int d^2k_1\psi(\mathbf{k}_1)\ln\xi^{-1/3} \propto -\Omega^{-3}\ln\Omega\,, \quad (13)$$

where $\Gamma(x)$ is Gamma function. The amplitude of density (11) has the same scaling ($\Omega^{-2/3}$) as the amplitude of passive scalar field χ with delta-correlated random velocity,[4] as it should. However, the density flux in Eq. (10), which is proportional to $\Omega^{-2}\ln\Omega$, is reduced much more than the passive scalar field flux $\langle \chi v_x \rangle \propto \Omega^0$. It is because the velocity amplitude $\langle v_x^2 \rangle \propto \Omega^{-3}$ is severely reduced by shearing (see Eq. (12)) in this dynamical model. Thus, as noted previously, here the reduction in the velocity amplitude is the most important factor which lowers the overall transport. In contrast, the cross-phase $\delta = \langle n v_x \rangle / \sqrt{\langle n^2 \rangle \langle v_x^2 \rangle} \propto \Omega^{-1/6}\ln\Omega$ is only weakly reduced by strong shear, with only a slight difference from the passive scalar field case ($\langle \chi v_x \rangle / \sqrt{\langle \chi^2 \rangle \langle v_x^2 \rangle} \propto \Omega^{1/6}$). This result is in a sharp contrast to Ref. 5. Note that the divergence of $\langle n^2 \rangle$ as $\xi \to 0$ is a consequence of the transport of density from large to small scales. Finally, Eq. (13) represents the Reynolds stress $\langle v_x v_y \rangle = -\nu_T \partial_{xx}\phi_0$, which drives a mean flow as $\partial_t(\partial_{xx}\phi_0) = -\partial_x \langle v_x v_y \rangle = \partial_x(\nu_T \partial_{xx}\phi_0)$. Here, ν_T is the turbulent viscosity and ϕ_0 is the mean electric potential ($\Omega = -\partial_x U_0 = -\partial_{xx}\phi_0$). That is, the turbulent viscosity is negative with its value $\nu_T \propto -\Omega^{-4}\ln(\Omega/Dk^2)$, representing the generation of a mean flow by the inverse cascade (e.g., see Ref. 10). In addition, this also shows that Reynolds stress driving for the mean flow itself is reduced as shearing becomes strong because of the damping of turbulence.[11] Note here that the momentum flux does not include an additional contribution from mean density gradient since that latter was treated as a part of noise f.

In the second case (ii) where the noise has a finite correlation time τ_f ($> \tau_\Omega$), we assume, for simplicity, that it has Lorenzian frequency spectrum centered at frequency ω_k with spread $\gamma_k = 1/\tau_f < \Omega$ as $\tilde{\psi}(\mathbf{k}, t_1 - t_2) = \int_{-\infty}^{\infty} d\omega \exp[-i\omega(t_2 - t_1)]\psi(\mathbf{k})\gamma_k / [(\omega - \omega_k)^2 + \gamma_k^2]$. Omitting intermediate steps, which turn out to be quite involved in this case, we here present final results only to leading order in $\xi = Dk_1^2/\Omega \ll 1$:

$$\langle n v_x \rangle \sim \frac{2\pi^2}{(2\pi)^2\Omega^3}\frac{g}{N_0}\int d^2k_1 \psi(\mathbf{k}_1)\delta(\varpi_k + k_{1y}x)\ln\xi^{-1/3} \propto \Omega^{-3}\ln\Omega\,, \tag{14}$$

$$\langle n^2 \rangle \sim \frac{2\pi^2}{(2\pi)^2\Omega^2}\int d^2k_1 \psi(\mathbf{k}_1)\delta(\varpi_k + k_{1y}x)\frac{1}{3}\Gamma\left(\frac{1}{3}\right)\left(\frac{3}{\xi}\right)^{1/3} \propto \Omega^{-5/3}\,, \tag{15}$$

$$\langle v_x^2 \rangle \sim \frac{\pi}{4(2\pi)^2\Omega^4}\left(\frac{g}{N_0}\right)^2\int d^2k_1 \psi(\mathbf{k}_1)\left\{\left[e^{\alpha}E_i(-\alpha) + e^{-\alpha}E_i(\alpha)\right]^2 + \pi^2 e^{-2|\alpha|}\right\} \tag{16}$$

$$\sim \frac{\pi}{(2\pi)^2\Omega^4}\left(\frac{g}{N_0}\right)^2 \int d^2k_1\psi(\mathbf{k}_1)\left[\mathcal{C}+\ln|\alpha|\right]^2 \propto \Omega^{-4}\,, \tag{17}$$

$$\langle v_x v_y\rangle \sim -\frac{\pi^2}{2(2\pi)^2\Omega^4}\left(\frac{g}{N_0}\right)^2 \int d^2k_1\psi(\mathbf{k}_1)\left[4\delta\left(\overline{\omega}_k+k_{1y}x\right)\ln\xi^{-1/3}+\ln|\alpha|\right] \propto -\Omega^{-4}\ln\Omega\,. \tag{18}$$

Here, $\overline{\omega}_k = \omega_k/\Omega$, $\overline{\gamma}_k = \gamma_k/\Omega$, $\pi\delta\left(\overline{\omega}_k + k_{1y}x\right) = \lim_{\overline{\gamma}_k\to 0}\overline{\gamma}_k/[(\overline{\omega}_k + k_{1y}x)^2 + \overline{\gamma}_k^2]$, $\alpha = \overline{\omega}_k + k_{1y}x - i\overline{\gamma}_k$, $E_i(x)$ is the exponential integral, and $\mathcal{C} = 0.57725$ is the Euler's constant. Note that the last term in Eq. (16) should be replaced by 0 when $\alpha = 0$. Since the exponential integral $E_i(\pm\alpha)$ in Eq. (16) becomes large for small argument α, $E_i(\pm\alpha)$ was expanded for small α as $E_i(\pm\alpha) \sim \mathcal{C} + \ln|\alpha| + O(\alpha)$ to obtain Eq. (17) and also Eq. (18). $\delta\left(\overline{\omega}_k + k_{1y}x\right)$ appearing in Eqs. (14), (15) and (18) explicitly shows the resonance effect where Doppler shifted frequency vanishes locally (i.e., $\omega_k - U_0k_{1y} = 0$). It is amusing to see that the resonance effect is regularized by dissipation in these three cases in Eqs. (14), (15) and (18), which diverge as $D(\propto\xi)\to 0$. In contrast, the resonance is replaced by a logarithmic singularity as $\omega_k - U_0k_{1y}\to 0$ for radial velocity amplitude in Eq. (17), where the divergence can now be regularized by the finite spread of noise frequency spectrum γ_k $(> Dk^2)$. This logarithmic singularity seems to originate from the resonance broadening in the presence of shear flow, as the latter keeps generating finer radial scales (i.e., large k_x), thereby causing time-transient (i.e., non-modal) behavior. For a given ω, the radial velocity v_x is very vulnerable to this transient effect (i.e. the increase in k_x) since $\hat{v}_x \propto \hat{\omega}/k^2 \sim \hat{\omega}/k_x^2$.

Compared to the short correlated noise, all the correlation functions in Eqs. (14), (15), (17), and (18) are reduced by one more power of Ω. The stronger effect of shear flow is expected due to its coherent shearing over time $t < \tau_f$, as previously noted. However, similar trend still persists in this case. First, severe reduction in the velocity amplitude ($\langle v_x^2\rangle \propto \Omega^{-4}$) is largely responsible for strong reduction in the particle transport $\langle nv_x\rangle \propto \Omega^{-3}\ln\Omega$. Note that this reduction is stronger than that for the passive scalar transport by a random velocity with finite frequency spectrum (i.e., $\langle\chi v_x\rangle \propto \Omega^{-1}$). Second, the cross-phase $\delta = \langle nv_x\rangle/\sqrt{\langle n^2\rangle\langle v_x^2\rangle}$ has the same weak scaling $\propto \Omega^{-1/6}\ln\Omega$ with Ω. Finally, turbulent viscosity from Reynolds stress is again negative with $\nu_T \propto -\Omega^{-5}\ln(\Omega/Dk^2)$, indicating the generation of mean flow from turbulence. These results are summarized in Table I.

III. HEAT TRANSPORT IN ITG TURBULENCE MODEL

The interchange turbulence model, despite its simplicity, revealed the importance of dynamical properties of the flow in determining the transport level. That is, the flow undergoes amplitude suppression due to mean flow shearing, leading to a stronger reduction in the particle flux ($\langle n v_x \rangle$), down to ($\propto \Omega^{-3} \ln \Omega$). While the exact scaling of the transport with shearing rate will certainly depend on the precise details of the turbulence model and also the quantity that is transported, it may yet be possible that the variation in scalings is rather small among different models, so long as they contain the same essential physical effects. If this were the case, the identification of the latter will be of great interest. One of such properties in our interchange model (Eqs. (3)-(4)) is that the flow is driven by density gradients coupled to unfavorable magnetic curvature, while the latter by a random noise in addition to density gradient. A similar dynamics of a flow (thus, having similar scaling for transport) may be found in other models. As an example, we now consider heat transport in a simple ITG turbulence in 2D slab geometry, which is driven by pressure gradient ($\partial_x p_0$) in a region with a unfavorable magnetic curvature ($(\partial_x p_0)(\partial_x B_0) < 0$):

$$(\partial_t + \mathbf{u} \cdot \nabla) p = -v_x \partial_x p_0 + \chi \nabla^2 p + f \,, \tag{19}$$

$$(\partial_t + \mathbf{u} \cdot \nabla)(1 - \nabla^2)\phi = -v_B \partial_y p - \nu \nabla^2 \nabla^2 \phi \,. \tag{20}$$

Here, a flat density profile is assumed, for simplicity. p_0 and p are mean and fluctuating components of pressure; $v_B \propto -\partial_x B_0$ represents the effective force due to magnetic curvature; ν and χ are viscosity and thermal diffusivity, respectively; f is the external pressure source. The free energy source in this model is the background pressure gradient term $v_x \partial_x p_0$, equivalent to $v_x \partial_x N_0$ in the interchange turbulence. Thus, by treating this free energy source, as well as incoherent nonlinear interaction, as a part of the total noise f, this ITG turbulence model becomes almost isomorphic to the interchange model upon changing (a) $p \to n$ and (b) $(1 - \nabla^2)\phi \to \omega$, with a difference in $\mathbf{v}$ in the two models: e.g., $\hat{v}_x = -ik_y[(1+k^2)\hat{\phi}]/(1+k^2)$ in ITG (which would correspond to $ik_y \hat{\omega}/(1+k^2)$ in interchange turbulence), while $\hat{v}_x = ik_y\hat{\omega}/k^2$ in the interchange model. This difference arises from the enhanced inertia in the ITG model, due to adiabatic electrons. It can, however, be shown that in the limit $\tau_\Omega \ll \tau_D$, this difference can be absorbed in parameters such as D, ω_k, γ_k, etc, without altering the overall scaling with shearing rate. Note that $\nu\nabla^2(1-\nabla^2)\phi \sim -\nu\nabla^2\nabla^2\phi$ because of fine radial scales k_x generated by shearing. Therefore, the scalings of $\langle pv_x\rangle$, $\langle p^2\rangle$, $\langle v_x^2\rangle$, and $\langle v_x v_y\rangle$ with Ω in ITG are similar to those of $\langle nv_x\rangle$, $\langle n^2\rangle$, $\langle v_x^2\rangle$, and $\langle v_x v_y\rangle$ in interchange turbulence. Note though that the turbulent viscosity in ITG should include an additional pressure driving (besides the Reynolds stress) due to pressure perturbation.

IV. CONCLUSION

We have studied the reduction in particle and heat transport by a mean shear flow in interchange and ITG turbulence models, respectively. In contrast to scalar field transport, turbulent flow (or, electric potential) evolves dynamically, subject to shearing by a mean shear flow in both models. We have shown that a strong reduction in the transport of particles (from interchange turbulence) and heat (from ITG turbulence) results from a severe reduction in the amplitude of velocity in both models. The very strong transport scaling with respect to the shearing rate ($\propto \Omega^{-3}\ln\Omega$) offers an explanation of some experimental results that the ion heat transport gets reduced to the level of neoclassical value due to the $\mathbf{E}\times\mathbf{B}$ shear[1] without a need to invoke an ad-hoc, but popular turbulence quenching rule involving the linear growth rate. Note that the scaling $\Omega^{-3}\ln\Omega$ of the flux is much stronger than that in the passive scalar field case ($\langle\chi v_x\rangle \propto \Omega^{-1}$).[4] Interestingly, this indicates the possibility of non-power-law scaling with Ω.

However, the reduction in cross-phase is very weak ($\delta \propto \Omega^{-1/6}\ln\Omega$). *That is, in our model, the transport is suppressed mainly through the reduction in the amplitude of turbulence, especially, that of the velocity.* This prediction is in semi-quantitative agreement with the recent results from gyrofluid simulations of toroidal ITG turbulence[6] where no relevant variation of the cross-phase is observed. It is worthwhile to note that the same conclusion can also be drawn[18] from the gyrokinetic simulations of toroidal ITG turbulence where a remarkable proportionality between ion heat transport and fluctuation intensity is observed to be maintained during the bursting phase.[12] Our results disagree with results from a previous computational study[7] and the claims of a very strong reduction in Ref. 5. Note though that a stronger reduction in the velocity amplitude than the density (temperature) in interchange (ITG) turbulence may be due to our treatment of the free energy source as a part of the random noise which drives the density (temperature) fluctuation, only. As a result, a direct coupling between the flow and density (heat) is lost, and a random noise acts only on density (heat). Thus, while this assumption is useful to understand the essential physics in the simplest level, it will be certainly interesting to study these models in a more self-consistent way.

Other important ingredients, which are yet missing in these dynamical models, are (i) the effect of zonal flow shearing and (2) the effect of shearing on intermittent transport. Throughout the discussion in the paper, shearing was taken to be coherent over the time interval of interest, with shear flows being implicitly assumed to be mean $\mathbf{E} \times \mathbf{B}$ flows. It is now, however, well-known that in addition to this mean $\mathbf{E} \times \mathbf{B}$ flow, shearing by self-generated zonal flows, with complex spatial structure and finite time correlation τ_{ZF}, is very effective in regulating turbulence.[12] They are likely to play a more important role before the L-H transition than a mean flow is, thereby possibly triggering the transition before the mean flow shearing (due to steepened pressure profile) becomes dominant and maintains the plasma in the H-mode after the L-H transition.[13,14] Thus, it is interesting to study the shearing effect of zonal flows. Of particular interest is the scaling of transport as τ_{ZF}/τ_{Ω} varies.[15] Secondly, in the case where transport in L mode is dominated by intermittent transport, associated with coherent structures (e.g., streamers, blobs, etc),[16] the effect of shearing on coherent structures themselves should be examined. Note that the evolution of the gradient of vorticity may depend on both strain and vorticity, with the possibility of vorticity inhibiting the generation of vorticity gradient.[17] These issues are under investigation and will be addressed in future papers.

Acknowledgments We thank X. Garbet, S.-I. Itoh, and K. Itoh for useful discussions. This research was supported by U.S. DOE FG03-88ER 53275 and by U.S. DOE Contract No. DE-AC02-76-CHO-3073.

REFERENCES

[1] K.H. Burrell, Phys. Plasmas **4**, 1499 (1997).

[2] R.R. Weynants, *et al*, Plasma Phys. Control. Fusion **40**, 635 (1998).

[3] J. Boedo, *et al*, Phys. Rev. Lett. **84**, 2630 (2000); J. Boedo, *et al*, Nuclear Fusion **42**, 117 (2002).

[4] E. Kim and P.H. Diamond, Phys. Rev. Lett. **91**, 075001 (2003).

[5] P.W. Terry, D.E. Newman, and A.S. Ware, Phys. Rev. Lett. **87**, 185001 (2001).

[6] G.L. Falchetto and M. Ottaviani, Phys. Rev. Lett. 92, 025002 (2004).

[7] C.F. Figarella, *et al*, Phys. Rev. Lett. **90**, 015002 (2003).

[8] E. Kim and B. Dubrulle, Phys. Plasmas **8**, 813 (2001); E. Kim and B. Dubrulle, Physica D **165**, 213 (2002); S. Nazarenko, *et al*, Physica D **139**, 158 (2000).

[9] P. Goldreich and D. Lynden-Bell, MNRAS **130**, 125 (1965).

[10] P.H. Diamond, *et al,* in *Plasma Phys. and Controlled Nuclear Fusion Research* (IAEA, Vienna, 1998) IAEA-CN-69/TH3/1.

[11] M. Malkov, *et al*, Phys. Plasmas **8**, 5073 (2001).

[12] Z. Lin, *et al*, Phys. Rev. Lett. **83**, 3645 (1999).

[13] E. Kim and P.H. Diamond, Phys. Rev. Lett. **90**, 185006 (2003).

[14] R.A. Moyer, *et al*, Phys. Rev. Lett. **87**, 135001 (2001); P.H. Diamond, *et al*, Phys. Rev. Lett. **84**, 4842 (2000).

[15] E. Kim and P.H. Diamond, Phys. Rev. Lett. submitted (2004).

[16] P.A. Politzer, Phys. Rev. Lett. **84**, 1192 (2000); P. Beyer, *et al,* Phys. Rev. Lett. **85**, 4892 (2000); J.F. Drake, *et al,* Phys. Rev. Lett. **61**, 2205 (1988).

[17] J.C. McWilliams, J. Fluid Mech. **146**, 21 (1984); J. Weiss, Physica D **48**, 273 (1991).

[18] T.S. Hahm, Plasma Phys. Control. Fusion, **44**, A87 (2002).

TABLES

TABLE I. Summary of scalings

	$\tau_f < \tau_\Omega$	$\tau_f > \tau_\Omega$
$\langle nv_x \rangle$ / $\langle pv_x \rangle$	$\Omega^{-2} \ln \Omega$	$\Omega^{-3} \ln \Omega$
$\langle n^2 \rangle$ / $\langle p^2 \rangle$	$\Omega^{-2/3}$	$\Omega^{-5/3}$
$\langle v_x^2 \rangle$	Ω^{-3}	Ω^{-4}
$\langle v_x v_y \rangle$	$-\Omega^{-3} \ln \Omega$	$-\Omega^{-4} \ln \Omega$
$\cos \delta$	$\Omega^{-1/6} \ln \Omega$	$\Omega^{-1/6} \ln \Omega$