Self Organization of Zonal Flows:

Some Physics Behind the Color Viewgraphs

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Abstract

Two model calculations which elucidate the basic physics of zonal flow generation are presented. The first uses statistical ray optics to clarify the physics of the modulational instability process. The second uses the envelope formalism to obtain and solve a simple system of equations for the zonal flow and drift wave envelope. This simple system is closely analogous to the Zakharov equations for Langmuir turbulence.

Keywords:

zonal flow, modulational instability, drift wave turbulence

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I. Introduction

Zonal flows are shear layers or strongly anisotropic vortices with $k_\theta = 0$, $k_{||} \cong 0$, $k_\perp \rho$, finite and (nearly) zero frequency, $\Omega \cong 0$. Since $k_\theta = 0$, $\tilde{V}_r = 0$, so that zonal flows are intrinsically incapable of driving transport, and thus represent a reservoir of benign fluctuation energy. Zonal flows may be thought of as a particular limit $(k_r >> k_\theta = 0)$ of the more general convective cell structure. In contrast to the familiar mean $E \times B_0$ shear flows, with direction determined by profile structure and characterized by a single scale, zonal flows are of indeterminate direction and exhibit a spectrum of scales producing a spatially complex flow profile. Zonal flows are nonlinearly generated by drift waves via modulations of the radial flux of vorticity (i.e. charge separation current [1,2]) and are damped by ion-ion collisions [3], nonlinear feed-back on the underlying drift waves [4] or (possibly) by Kelvin-Helmholtz type instabilities which disrupt them [5].

Several aspects of zonal flows mentioned above have been extensively (indeed exhaustively) studied, especially via numerical simulations. Such simulations are powerful tools, and yield results which can be attractively packaged as striking color viewgraphs, for example. However, there is still a need for physical insight, as gleaned from simplified, model problems. Moreover, such model problems often prove useful in benchmarking complex codes. In this paper, we discuss two model calculations pertinent to zonal flow physics. In Section II, a physical picture of zonal flow generation, using coherent and statistical ray theory, is presented. This picture confirms and elucidates results obtained previously using modulational analyses. In Section III, envelope perturbation theory is used to derive a "minimal" system of equations for the drift wave

+ coherent flow problem. This system is analogous to the Zakharov equations familiar from Langmuir turbulence, and may be used to examine both zonal and streamer flow growth.

II. Physical Picture of Zonal Flow Generation

It is illuminating to present a short, 'back-of-an-envelope' type demonstration of zonal flow instability. Consider a packet of drift waves propagating in an ensemble of quasi-stationary, random zonal flow shear layers. Take the zonal flows as slowly varying with respect to the drift waves (i.e. $\Omega << \omega_{\underline{k}}$), etc. quasi-stationary. The *spatially complex shearing flow will result in an increase in* $\langle k_r^2 \rangle$, the mean-square radial wave vector (i.e. consider a random walk of k_r, as described by eikonal theory). In turn the drift wave frequency $\omega_e * / \left(1 + k_\perp^2 \rho_s^2\right)$ must then *decrease*. Since $\Omega << \omega_{\underline{k}}$, the drift wave action density $N(\underline{k}) = \varepsilon(\underline{k}) / \omega_{\underline{k}}$ is conserved, so that *drift wave energy must also decrease*. As the total energy of the system of waves and flows is also conserved (i.e. $\varepsilon_{wave} + \varepsilon_{FLOW} = const.$), it thus follows that *the zonal flow energy must*, in turn, *increase*. Hence, the initial perturbation is re-enforced, suggestive of instability. Note that the simplicity and clarity of this argument supports the assertion that zonal flow generation is a robust and ubiquitous phenomenon.

A slightly larger envelope is required for a 'physical argument' which is also quantitatively predictive. Consider a drift wave packet propagating in a sheared flow field. Take $\omega_{\underline{k}} > \left|V_E'\right|$ and $|\underline{k}| > \left|V_E' / V_E\right|$, so that wave action density is conserved (i.e. $N(k) = N_0$, a constant). Thus, wave energy density evolves according to:

$$\frac{d}{dt}\varepsilon(\underline{k}) = N_0 \frac{d\,\omega_{\underline{k}}}{dt}$$

$$= N_0 \left(\frac{\partial \omega_{\underline{k}}}{\partial t} + \underline{V}_g \frac{\partial \omega_{\underline{k}}}{\partial \underline{x}_x} + \frac{\partial \omega_{\underline{k}}}{\partial k_{\underline{x}}} \cdot \frac{d\,k_{\underline{k}}}{dt} \right) \qquad (1)$$

$$\cong \left(\frac{2\,k_r k_\theta \rho_s^2}{1 + k_\perp^2 \rho_s^2} \right) V_E' \varepsilon(\underline{k}).$$

Here, we have assumed stationary, isotropic turbulence and have used the eikonal equation $d\,k_r/dt = -\partial(k_\theta V_E)/\partial x$. Eqn. (1) just states that the drift wave packet loses or gains energy via work on the mean flow via wave-induced Reynolds stress. (Indeed, $k_r k_\theta \varepsilon(\underline{k}) \langle \tilde{V}_r \tilde{V}_\theta \rangle$!). Note as well that the factor $k_r k_\theta \varepsilon(\underline{k}) V_E'$ is rather obviously suggestive of the role of triad interactions in controlling fluctuation - flow energy exchange. For zonal flows, the shear is random and broad-band, so that $V_E' \to \tilde{V}_E$, $N \to \langle N \rangle + \tilde{N}$, and $N_0 V_E' \to \left\langle \tilde{N} \tilde{V}_E' \right\rangle$. Hence, Eqn. (1) may be re-written as:

$$\frac{d}{dt}\varepsilon(\underline{k}) = -V_{g,r} k_\theta \left\langle \tilde{V}_E' \tilde{N} \right\rangle. \qquad (2)$$

To complete the argument, the correlator $\left\langle \tilde{V}_E' \tilde{N} \right\rangle$ must be calculated. To this end, we use the Wave Kinetic Equation (W.K.E.)

$$\frac{\partial N}{\partial t}+\left(\underline{V}_g+\underline{V}\right)\cdot\underline{\nabla}N-\frac{\partial}{\partial\underline{x}}(\omega+k_\theta V_E)\cdot\frac{\partial N}{\partial\underline{k}}=\gamma_{\underline{k}}N-\Delta\omega_{\underline{k}}N^2/N_0, \tag{3}$$

and the methodology of quasilinear theory to obtain:

$$k_\theta\left\langle\tilde{V}_E'\tilde{N}\right\rangle=D_K\frac{\partial\langle N\rangle}{\partial k_r}, \tag{4a}$$

$$D_K=k_\theta^2\sum_q q^2\left|\tilde{V}_{Eq}'\right|^2 R(\underline{k},q), \tag{4b}$$

$$R(\underline{k},q)=\gamma_{\underline{k}}\Big/\left(\left(qV_{g,r}\right)^2+\gamma_{\underline{k}}^2\right). \tag{4c}$$

The term $\Delta\omega_{\underline{k}}N^2/N_0$ represents drift wave nonlinear damping via self-interaction of drift waves (i.e. cascade by *local* interaction).

Here q is the radial wave number of the zonal flow and equilibrium balance in the absence of flow has been used to relate $\Delta\omega_{\underline{k}}$ to $\gamma_{\underline{k}}$. The wave energy then evolves according to:

$$\frac{d\varepsilon(\underline{k})}{dt}=\frac{2\rho_s^2 D_K k_r}{\left(1+k_\perp^2\rho_s^2\right)^2}\ \frac{\partial\langle N\rangle}{\partial k_r}. \tag{5}$$

As total energy of the stationary wave-flow system is conserved, $d/dt\left(\sum_{\underline{k}}\varepsilon(\underline{k})+\sum_q\left|\tilde{V}_q\right|^2\right)=0$. Thus, the zonal flow generation rate is determined to be:

$$\gamma_q = -2q^2 c_s^2 \sum_{\underline{k}} \frac{k_\theta^2 \rho_s^2}{\left(1 + k_\perp^2 \rho_s^2\right)^2} R\left(\underline{k}, \underline{q}\right) k_r \frac{\partial}{\partial k_r} \langle \eta \rangle, \tag{6a}$$

$$\langle \eta \rangle = \left(1 + k_\perp^2 \rho_s^2\right) \langle \varepsilon \rangle . \tag{6b}$$

Here $\langle \eta \rangle$ is the mean potential enstrophy density of the drift wave turbulence.

The result given above in Eqn. (6a), obtained by transparent physical reasoning, is identical to that derived previously by formal modulational stability arguments [6]. Note that $\partial \langle \eta \rangle / \partial k_r < 0$ (a condition which is virtually always satisfied in drift wave turbulence) is required for zonal flow growth. In addition, the argument above reveals that drift wave *ray chaos* provides the key element of irreversibility, which is crucial to the wave-flow energy transfer dynamics. Here, ray chaos requires overlap of the $\Omega/q = V_{g,r}$ resonances in D_k, a condition easily satisfied for finite lifetime drift wave eddys and (nearly) zero frequency zonal flows (i.e. $\Delta \omega_{\underline{k}} >> \Omega$). Under these conditions a positive Lyapuanov exponent is present and neighboring drift waves rays diverge exponentially in time. Ray chaos in turn assures that zonal flow shearing and wave refraction are *random*, thus validating the use of the stochastic methodology employed here. In the case where rays are *not* chaotic, envelope perturbation formalism, methods from the theory of trapping, or parametric instability theory must be used to calculate zonal flow generation.

Recent gyrokinetic simulations have demonstrated that modulational instability growth of zonal flow perturbations in fully developed drift-ITG turbulence can occur. In particular, recent simulation results show that the evolution of a (seed) zonal flow perturbation initialized in a bath of quasi-stationary ITG turbulence which had already saturated by other processes. The seed perturbation clearly grows exponentially and its growth induces a further decrease in the ion thermal diffusivity, as measured by the simulation. Nevertheless, this evidence for the viability of the modulational instability of zonal flows in turbulence, as well as their role in regulating transport, is quite compelling.

III. Envelope Equation Theory of Zonal Flow and Streamer Generation

In this section we explore the dynamics of streamer and zonal $(k_{\|}=0)$ flow generation by examining the stability of envelope modulations to a drift wave packet. Recent simulations [7] suggest that streamers are quasi-flute nonlinear structures, with $0 \lessapprox k_{\|} << 1/Rq$. We focus then on $k_{\|}=0$ large scale flow generation. Reductive perturbation theory is applied to the coupled Hasegawa-Mima [8] equation for drift waves including a modified plasma response to the modulations of the density and electrostatic potential Eqn. (7) and the 2D Euler equation Eqn. (8) (equivalent to the mean polarization current) and continuity equation Eqn. (9) for the mean flow. This system describes $k_{\|}=0$ flow and drift wave envelope structures with arbitrary anisotropy in the poloidal plane, and addresses both the streamer and zonal flow formation problems on an equal footing. The envelope and $k_{\|}=0$ mean flow equations we obtain constitute the counterpart for the drift wave problem of the well known Zakharov equations for Langmuir turbulence [9]. Indeed, the processes of zonal flow and streamer formation may be viewed as different manifestations of a nonlinear 'self-shearing' instability process, which is somewhat akin to self-focusing or caviton formation. This investigation also complements previous works on density and temperature gradient drift wave soliton within adiabatic regime [10 ,11], as well as previous theoretical work on zonal flows, which was based on wave kinetics and eikonal theory [12] or on a model parametric instability analysis in zonal flow collisional regime [13].

In order to describe nonlocal interactions in drift-wave turbulence, the approach adopted here is based on a two scale analysis where the density and electrostatic potential fields are split into small scale $\widetilde{(\cdot)}$ and large scale $\overline{(\cdot)}$ fluctuations $n = \tilde{n} + \overline{n}$ and $\phi = \tilde{\phi} + \overline{\phi}$. The small scale turbulence is considered within the adiabatic regime $\alpha = v_{the}^2 k_\parallel^2 / \nu \omega_0 > 1$ with α the parallel electron diffusion in a characteristic wave period ω_0. In this regime, electrons tend to a Boltzmann distribution $\tilde{n} = e\tilde{\phi} / T_e$. In contrast, the large scale fields associated here to the limit $\alpha \to 0$ correspond to $k_\parallel = 0$ modes. The Boltzmann response then does not hold on large scale, (i.e. there $\overline{n} \neq e\overline{\phi} / T_e$. Starting from the Hasegawa-Wakatani description [14], the appropriate equations to describe the drift-waves dynamics in presence of mean flow have the form:

$$\partial_t \left(\tilde{n} - \rho_s^2 \Delta_\perp \frac{e}{T_e} \tilde{\phi} \right) + \mathrm{v}^* \partial_y \frac{e}{T_e} \tilde{\phi}$$

$$+ \overline{\mathrm{v}}_E \cdot \nabla \left(\tilde{n} - \rho_s^2 \Delta_\perp \frac{e}{T_e} \tilde{\phi} \right) + \tilde{\mathrm{v}}_E \cdot \nabla \left(\overline{n} - \rho_s^2 \Delta_\perp \frac{e}{T_e} \overline{\phi} \right) = 0 , \qquad (7)$$

$$\partial_t \Delta_\perp \overline{\phi} + \overline{\tilde{\mathrm{v}}_E \cdot \nabla \Delta_\perp \tilde{\phi}} = 0 , \qquad (8)$$

$$\partial_t \overline{n} + \mathrm{v}^* \partial_y \frac{e}{T_e} \overline{\phi} + \overline{\tilde{\mathrm{v}}_E \cdot \nabla \tilde{n}} = 0 . \qquad (9)$$

where the electron temperature T_e is assumed constant. Here notation is standard, i.e., $L_n = -\left(d_X n_0 / n_0\right)^{-1}$ the logarithmic density length scale, $\mathrm{v}^* = \rho_s^2 \Omega_i / L_n$, $\rho_s^2 = T_e / \left(m_i \Omega_i^2\right)$ and $\mathrm{v}_E = \rho_s^2 \Omega_i \hat{z} \times \nabla e\phi / T_e$ is the $E \times B$ drift velocity with ***z*** direction of the magnetic field. In a quasi-neutral plasma, Eqns. (7)-(9) simply derive from the sum and difference of ion and electron continuity equations. Note that the mean-field $E \times B$ advection of ***n*** is retained in Eqn (7), since $\bar{n} \neq e\bar{\phi}/T_e$ for the mean flow. Indeed this mean-field $E \times B$ advection incorporates the important effect whereby the drift waves are sheared by the driven secondary flow (i.e. zonal flow and streamer). The self-interaction (i.e. local-interaction) is not retained in Eqn. (7), this effect is indeed negligible when dealing with a narrow drift-wave packet. Eqn. (8) reflects the fact that since the mean field perturbations (i.e. streamers and zonal flows) have $k_{\parallel} = 0$, the condition $\nabla \cdot J = 0$ reduces to $\nabla \cdot J_{pol} = 0$, identical to the 2D Euler equation.

In the search for identification of a mechanism for large scale structures generation, we focus on the modulation of monochromatic drift-wave propagating in the poloidal plane $\tilde{n} = N e^{i(k \cdot x - \omega t)} + c \cdot c$, $\tilde{\phi} = \Phi e^{i(k \cdot x - \omega t)} + c \cdot c$, with $N = e\Phi / T_e$ and where $c \cdot c$ stands for the complex conjugate. The frequency ω and wavevector $k = \left(k_x k_y\right)$ satisfies the dispersion relation $\omega = v^* k_y / \left(1 + \rho_s^2 k^2\right)$ and $k^2 = k_x^2 + k_y^2$. The drift-wave envelope *N(X,T)* is assumed slowly varying in time and space on the variation scales of the secondary flow field $\left(\bar{n}, \bar{\phi}\right)$. The long scales introduced with $T = \varepsilon t$ and ***X*** = *(X, Y)* = $(\varepsilon x, \varepsilon y)$, are as such that the variation of the background inhomogeneity on the modulation scales can be viewed as constant. We perform a reductive perturbative expansion [15] in powers of a small parameter ε measuring the relative magnitude of the wave amplitude, with $(\cdot) = (\tilde{\cdot}) + (\bar{\cdot}) = \varepsilon(\cdot)_1 + \varepsilon^2(\cdot)_2 + \ldots$. The drift-wave envelope is found to be affected, on time scale of order $1/\varepsilon^2$, by diffraction and nonlinear coupling to secondary mean flows. In the reference frame moving at the group velocity, the dynamics is governed by

$$i\,\partial_\tau N + \frac{1}{2}\left(\frac{\partial^2\omega}{\partial k_x^2}\partial_{XX} + \frac{\partial^2\omega}{\partial k_y^2}\partial_{YY} + 2\frac{\partial^2\omega}{\partial k_x \partial k_y}\partial_{XY}\right)N$$

$$+\rho_s^2\Omega_i\left(k\times\nabla\frac{e\bar{\phi}}{T_e}\right)\cdot\hat{z}N - \frac{\rho_s^2\Omega_i}{1+\rho_s^2k^2}\left(k\times\nabla\bar{n}\right)\cdot\hat{z}N = 0\,, \qquad (10)$$

$$\left(\varepsilon\partial_\tau - \mathrm{v}_g\cdot\nabla\right)\left(\partial_{XX}+\partial_{YY}\right)\frac{e\bar{\phi}}{T_e}$$

$$+2\rho_s^2\Omega_i\left(k_xk_y(\partial_{YY}-\partial_{XX}) + \left(k_x^2-k_y^2\right)\partial_{XY}\right)\left|N\right|^2 = 0\,, \qquad (11)$$

$$\left(\varepsilon\partial_\tau - \mathrm{v}_g\cdot\nabla\right)\bar{n} + \mathrm{v}^*\partial_Y\frac{e}{T_e}\bar{\phi} = 0, \qquad (12)$$

where $N = N_1 + \varepsilon N_2$, $\bar{\phi} = \bar{\phi}_1$, $\bar{n} = \bar{n}_1$ $\tau = \varepsilon T$, $\nabla = (\partial_X, \partial_Y)$. It, in turn, reacts back on the drift-wave envelope dynamics through nonlinear coupling via $E\times B$ sheared flow and polarization drift. Eqns. (10)-(12) constitute a unified 2D system which describes mean flow generation via *anisotropic* modulation of quasi-monochromatic drift-wave like streamers ($\partial_X \to 0$) and zonal flows ($\partial_Y \to 0$). Indeed Eqns. (10) and (12) are the direct analogues for the drift wave problem of the well known Zakharov equations for Langmuir Turbulence [16]. In what follows, we concentrate on the sub-celeric limit $\left(\varepsilon \partial_\tau \left(\overline{\cdot}\right) << \mathrm{v}_g \cdot \nabla \left(\overline{\cdot}\right)\right)$. This regime is analogous to the "subsonic" limit in Langmuir turbulence, where the acoustic perturbations are almost static and adiabatically follow the Langmuir wave modulation.

We first discuss streamer generation ($\partial_X \to 0$) in the sub-celeric limit. The streamer flow is then slaved to the drift-wave stress, and is given by $\partial_Y\left(e\overline{\phi}/T_e\right) = \left(2\Omega_i \rho_s^2 k_x k_y / \left(\partial\omega/\partial k_y\right)\right)\left|N\right|^2$ and $\overline{n} = \mathrm{v}^* / \partial\omega / \partial k_y e\overline{\phi} / T_e$. Thus the drift-wave envelope dynamics is governed by a cubic nonlinear Schrödinger (NLS) equation

$$i\partial_\tau N + \frac{1}{2}\frac{\partial^2 \omega}{\partial k_y^2}\partial_{YY} N$$

$$+\frac{2\Omega_i^2 \rho_s^4 k_x^2 k_\mathrm{y}}{\dfrac{\partial\omega}{\partial k_\mathrm{y}}}\left(1 - \frac{\mathrm{v}^*}{\dfrac{\partial\omega}{\partial k_\mathrm{y}}\left(1+\rho_s^2 k^2\right)}\right) N\left|N\right|^2 = 0. \qquad (13)$$

The NLS equation admits exact solutions of the form $N = N_0 exp\left(i\alpha_2\left|N_0\right|^2 \tau\right)$ with α_2 the nonlinear coefficient in Eqn. (13), which corresponds to a plane drift-wave with frequency slightly shifted by the nonlinearity. A stability analysis yields the dispersion relation $\Omega^2 = \alpha_1^2 Q_y^4 - 2\alpha_1\alpha_2\left|N_0\right|^2 Q_y^2$ where $2\alpha_1 = \partial^2\omega/\partial k_y^2$ and Ω and Q_y are the frequency and wavenumber associated with the phase and amplitude perturbations. A plane wave solution is unstable to large scale disturbances when the dispersive and nonlinear coefficients have the same signs $(\alpha_1\alpha_2 > 0)$, while smaller scales are stabilized by dispersion. The instability domain is thus defined by $\rho_s^2 k_y^2 < 3\left(1+\rho_s^2 k_x^2\right)$ as shown in Fig.1. As usual, the instability boundary is determined by the competition between diffraction and self-focusing (here 'focusing' corresponds to self-reinforcing shearing), so that modulations with $Q_y^2 > 2(\alpha_1/\alpha_2)\left|N_0\right|^2$ are stable. The streamer modulational instability growth rate $\gamma = \left|\mathrm{Im}(\Omega)\right|$ is maximal for $Q_y = Q_m$, such that

$$Q_m = \frac{2\Omega_i \rho_s^2 k_x k_y \left(1+\rho_s^2 k^2\right)^{\frac{5}{2}}}{3^{\frac{1}{2}} v^* \left(\rho_s^2 k_y^2 - \rho_s^2 k_x^2 - 1\right)\left|1+\rho_s^2 k_x^2 - \rho_s^2 k_y^2/3\right|^{\frac{1}{2}}}\left|N_0\right|, \tag{14}$$

The maximum growth rate is then

$$\gamma_m = \frac{4\Omega_i^2 \rho_s^6 k_x^2 k_y^2 \left(1+\rho_s^2 k^2\right)^2}{\left(\rho_s^2 k_y^2 - \rho_s^2 k_x^2 - 1\right)^2} \frac{\left|k_y\right|}{\left|\mathrm{v}^*\right|}\left|N_0\right|^2 . \tag{15}$$

Note that the most unstable scale Q_m^{-1} then gives a typical scale for the poloidal extent of the streamer. Indeed, the modulation undergoes a break up of the plane wave into a periodic pulse train, with Q_m^{-1} is approximately the width of a pulse envelope soliton of amplitude $\left|N_0\right|$ [17].

In the case of zonal flows $(\partial_X \to 0)$ generation, collisional damping of the poloidal flow must be taken into account [18]. For damping of poloidal flows with the form of a drag, $\tilde{\nu} = \varepsilon\nu$, where ν is the ion-ion collision frequency, the zonal flow equation then becomes

$$\left(\varepsilon\partial_\tau - \frac{\partial\omega}{\partial k_X}\partial_X + \nu\right)\partial_{XX}\frac{e\bar{\phi}}{T_e} - 2\rho_s^2\Omega_i k_x k_y \partial_{XX}|N|^2 = 0. \qquad (16)$$

Concentrating on the sub-celeric limit, advection and collisional damping compete to balance the nonlinear Reynolds stress drive. Two regimes must then be considered, according to the length scale at which the zonal flow is excited.

In the weakly-collisional regime, corresponding to a modulation wavenumber Q_x greater than $\nu/(\partial\omega/\partial k_X)$, the zonal flows are slaved to the drift-wave stresses by a collisionless response, i.e. $\partial_X\left(e\bar{\phi}/T_e\right) = -\left(2\Omega_i\rho_s^2 k_x k_y/(\partial\omega/\partial k_X)\right)|N|^2$. Note that, in contrast to the streamer case, the mean density is weaker $(\bar{n}_1 = 0)$ and its effect on drift wave dynamics is subdominant compare to the mean potential $\bar{\phi}$. As rather like the case of streamer generation, the drift-wave envelope then obeys a one-dimensional cubic NLS equation

$$i\partial_\tau N + \frac{1}{2}\frac{\partial^2\omega}{\partial k_x^2}\partial_{XX}N + \frac{2\Omega_i^2\rho_s^4 k_y^2 k_x}{\frac{\partial\omega}{\partial k_X}}|N|^2 N = 0. \qquad (17)$$

As previously, it can be straightforwardly shown that a plane drift-wave of amplitude N_0 is modulationally unstable for wavenumbers satisfying $1+\rho_s^2 k_y^2 - \rho_s^2 k_x^2 > 0$ [19]. The most unstable scale is given by Q_m^{-1}, where

$$Q_m = |N_0| \left(\frac{\Omega_i^2 \left(1+\rho_s^2 k^2\right)^5}{\mathrm{v}^{*2}\left(1+\rho_s^2 k_y^2 - \rho_s^2 k_x^2\right)} \right)^{1/2} . \qquad (18)$$

The associated maximal growth rate is estimated to be

$$\gamma_m = \Omega_i^2 \rho_s^2 \left(1+\rho_s^2 k^2\right)^2 \left|\frac{k_y}{\mathrm{v}^*}\right| |N_0|^2 . \qquad (19)$$

The zonal flow instability domain shown in Fig.1 appears to be quite extended in the (k_x, k_y) plane. It overlaps the instability domain for streamer generation defining a rather localized domain. It follows then than drift-waves whose wavenumbers satisfy the inequality $\rho_s^2 k_x^2 - 1 < \rho_s^2 k_y^2 < 3\left(1+\rho_s^2 k_x^2\right)$ can generate *both* kinds of anisotropic mean flows via modulational instability.

The other regime to investigate for zonal flow excitation corresponds to a collision dominated or dissipative limit [10], when the perturbation wavenumber Q_x is smaller than the ratio $\nu/(\partial\omega/\partial k_x)$. The zonal flow response to the drift wave stresses is then dissipative $e\overline{\Phi}/T_e = \left(2\rho_s^2 k_x k_y \Omega_i/\nu\right)|\tilde{N}|^2$. In turn, the drift-wave envelope obeys a NLS type equation with derivative nonlinearity

$$i\partial_\tau N + \frac{1}{2}\frac{\partial^2\omega}{\partial k_x^2}\partial_{XX} N - \frac{2\rho_s^4 k_y^2 k_x \Omega_i^2}{\nu} N \partial_X |N|^2 = 0 \qquad (20)$$

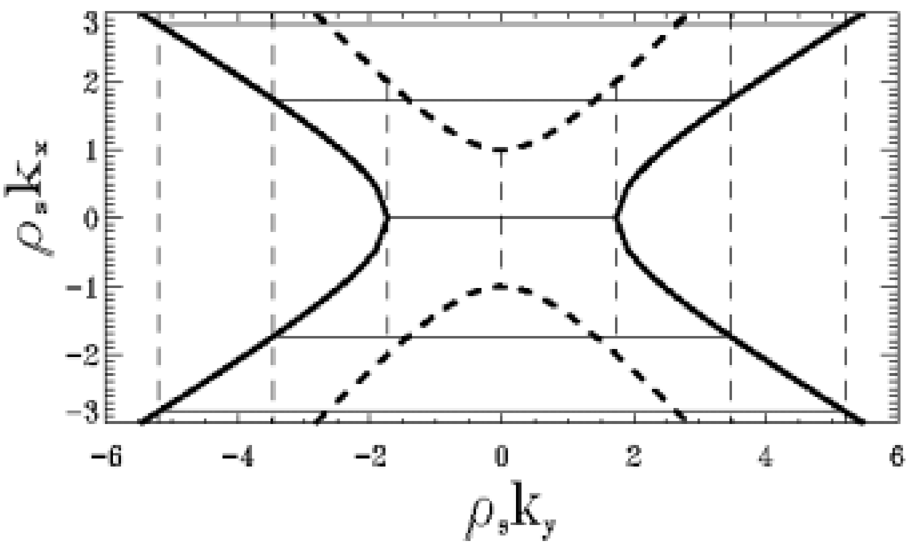

Figure 1 :
Instability domain for streamer generation (solid lines) and for non dissipative zonal flow generation (dashed lines) in the $(\rho_s k_y, \rho_s k_x)$ plane.

A stability analysis shows that a plane wave solution of amplitude N_0 is modulationally unstable to perturbations in phase and amplitude with frequency Ω and wavenumber Q_x satisfying the dispersion relation $\Omega^2 = \alpha_3^2 Q_x^4 + 2i\alpha_3\alpha_4 |N_0|^2 Q_x^3$, with $\alpha_3 = \left(\partial^2\omega/\partial k_x^2\right)/2$ and $\alpha_4 = 2\rho_s^4 k_y^2 k_x \Omega_i^2/\nu$. It turns out that a plane wave is unstable on *all* scales within the validity range of the envelope equation, a region limited by perturbation wavenumber Q_x being smaller than k/ε. The instability growth rate γ is given by $\gamma^2 = \frac{1}{2}\alpha_3^2 Q_x^4 \left(-1 + \left(1 + \left(2\alpha_4 |N_0|^2 / \left(\alpha_3 Q_x\right)\right)^2\right)^{\frac{1}{2}}\right)$. It is noticeable that the modulational instability grows faster on small scales at a rate of order $|\alpha_3|^{\frac{1}{2}} |N_0|^2 Q_x$, while on large scales the growth rate is reduced to order $|\alpha_3\alpha_4|^{\frac{1}{2}} |N_0| Q_x^{\frac{3}{2}}$. Thus, the case of dissipative dynamics turns out to be significantly different from that of weakly collisional zonal flow generation. The dynamics tied to this new kind of nonlinearity will be addressed in detail in a future publication.

One question of considerable interest concerns the relative importance of, and competition between, zonal flow and streamer structure formation. Comparison of typical scales and maximal growth rates associated with streamer $(\Omega_{s,} Q_s)$ and weak collisional zonal flow $(\Omega_{zf,} Q_{zf})$ generation gives $\Omega_s / Q_{zf} = 4\rho_s^4 k_x^2 k_y^2 / (\rho_s^2 k_y^2 - \rho_s^2 k_x^2 - 1)^2$, $(\Omega_s / Q_{zf})^2 = 4\rho_s^4 k_x^2 k_y^2 |1 + \rho_s^2 k_y^2 - \rho_s^2 k_x^2| / 3$ $(\rho_s^2 k_y^2 - \rho_s^2 k_x^2 - 1)^2 |1 + \rho_s^2 k_x^2 - \rho_s^2 k_y^2 / 3|$. These show that for drift-waves characterized by $\rho_s^2 k^2 << 1$, secondary zonal flows grow faster than streamer (see Fig.2a), while in the opposite limit $\rho_s^2 k^2 >> 1$, streamers are more easily excited by drift-wave modulation (see Fig. 2b). The typical poloidal and radial extent of streamers and zonal flows will depend on the strength of the dispersion in group velocity as compared to nonlinear effects. A comparison of the robustness of the nonlinear soliton structures formed by streamers and zonal flows with respect to radial and poloidal long-wavelength disturbances due respectively to growing zonal flows and streamers, is required to draw more far-reaching conclusions.

A second question concerns the scale and spatial structure of the streamer and zonal flow patterns. Note that the validity domain of the modulational analysis sits within spatial and temporal scales larger or the order of respectively $(\delta n / n_0)^{-1}$ and $(\delta n / n_0)^{-2}$. In the case of the streamers, Eqn. (13) suggests that the cells, which may be viewed as poloidally localized but radially extended solitons -see Fig. 3a, have poloidal extent $\Delta L_\theta \sim (\rho_s^2 / L_n)(\delta n / n_0)^{-1} F_s(\rho_s \mathrm{k})$,

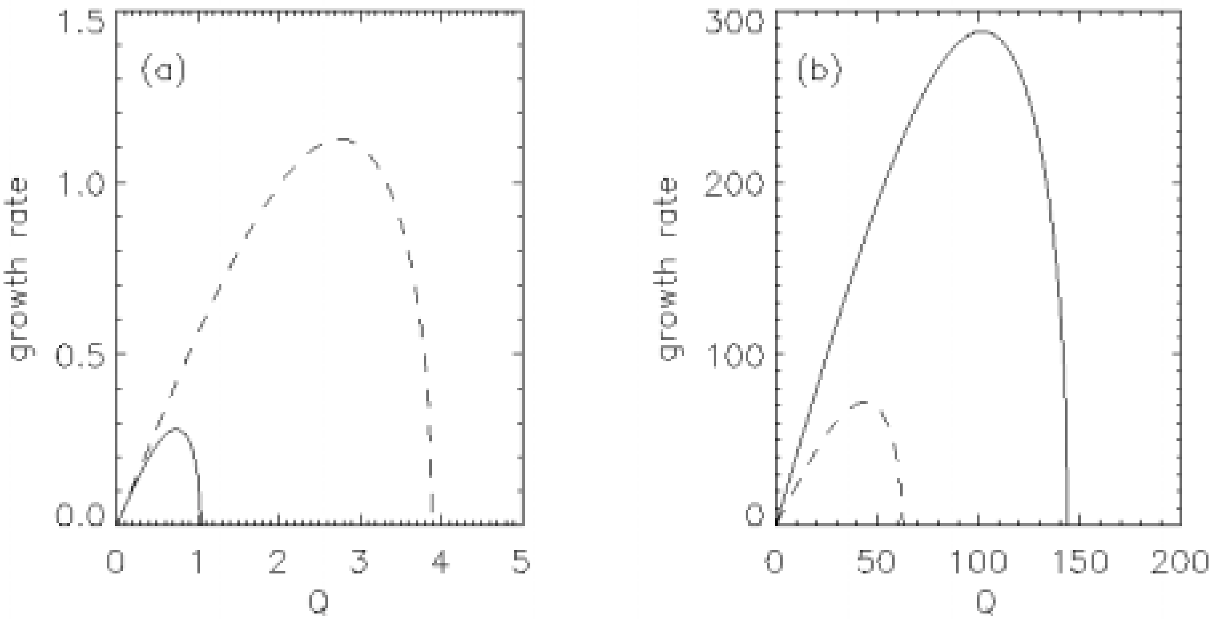

Figure 2:
Instability growth rate γ versus perturbation wavenumber Q associated to streamer (solid line) and non-dissipative zonal flow (dashed line) for $\rho_s = 1$, $v^* = 1$, $N_0 = 1$ and $\Omega_i = 1$, $k_x = 0.5$ and $k_y = 0.5$ panel (a) and for $k_x = 1$, $k_y = 2$ panel (b).

where $(\delta n/n_0)$ is the r.m.s. density fluctuation level of the drift wave, and $F_s(\rho_s \mathrm{k})$ contains information concerning dispersion, etc. Similarly, the maximal growth rate for streamers scales as Eqn. (14) $\gamma \sim (\delta n/n_0)^2 \Omega_i k_y L_n G_s(\rho_s \mathrm{k})$, so, for mixing length levels, streamer growth is quite virulent. For weakly collisional zonal flows (poloidally extended solitons -see Fig. 3b-) the radial extent is $\Delta L_r \sim (\rho_s^2/L_n)(\delta n/n_0)^{-1} F_{zf}(\rho_s \mathrm{k})$ (see Eqn. (18)). Results indicate that $\Delta L_r \sim \rho_s F_{zf}(\rho_s \mathrm{k})$ for $\delta n/n_0 \sim \rho_s/L_n$, so that a range of zonal flow scales can be expected, depending upon the dispersion properties of mode **k**. Note that both streamers and zonal flows will be strongly localized near caustics where $\partial^2 \omega/\partial k_i^2 \to 0$, a limit referred as the "semi-classical limit" and numerically investigated in [20]. Finite amplitude oscillations interpreted as a "sea of solitons" develop on intermediate scales shorter than the modulation scales, but still longer than the carrier wavelength.

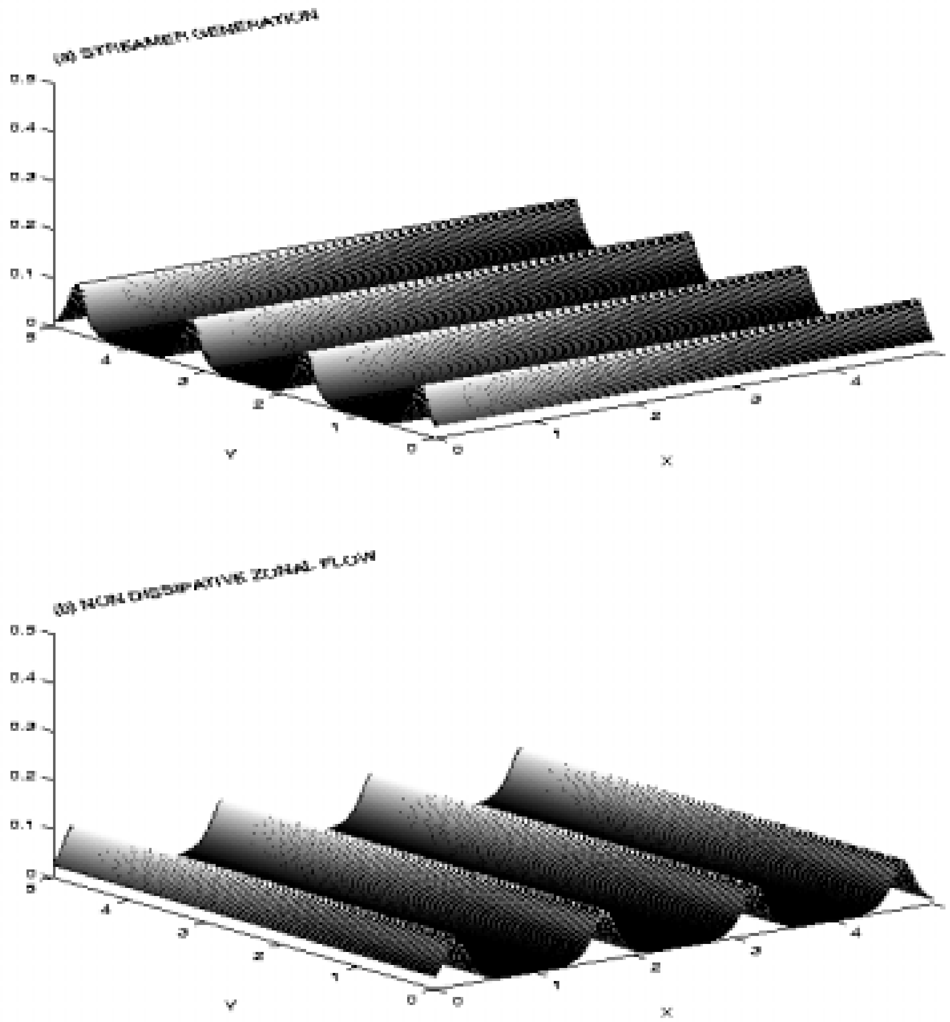

Figure 3 :
Cartoon of nonlinear structures associated with streamer (a) and zonal flow (b).

A third question concerns the ultimate nonlinear evolution and the structures. For the case of streamers, which are described by a one dimensional cubic NLS equation, a streamer wave-train modulation can be expected to break up [17] into train of radially extended solitonic type structures. Such streamer-induced solitons, then, are natural candidates for a dynamical realization of the 'avalanche' concept. Indeed very recent analyses [21] of numerical simulations indicate that during transport 'events' (i.e. avalanches), the radial correlation length of the *intensity* (i.e. spectrum envelope) of the participating drift modes significantly exceeds the correlation length of the drift wave eddy *potential*. This finding is thus consistent with the theory presented here, in which streamer formation is tied to envelope modulation. Similarly, weakly collisional zonal flow modulations can also be expected to break up into a train of soliton-like poloidal shear flow cells. In the case of strongly collisional zonal flow modulations, the nature of the nonlinear structure is not understood and must be await discussion in a future publication. Of course, noise excitation must be added in order to directly address many aspects of the fully *turbulent* state. Indeed, this investigation suggests that it may be profitable to reconsider the classic problem of drift wave turbulence as one cast in terms of the nonlinear interaction of filamentary structures [22]. Finally, one should keep in mind that tertiary instabilities, most notably those of the Kelvin-Helmholtz variety, may limit streamer and zonal flow lifetimes.

IV. Conclusion

In this paper, we have sought to elucidate certain elements of the basic physics of zonal flow dynamics. We have presented two simplified model calculations which support, clarify and elucidate previous results obtained by more formal methods. In the first of these (section II), statistical ray optics was used two describe zonal flow generation. In the second (Section III), envelope analysis techniques were used to derive a 2D, anistropic system akin to that of the Zakharov equations from Langmuir turbulence. These general equations were simplified to 1D Nonlinear Schrodiner equations for the case of zonal flows $\left(k_\phi = 0\right)$ and streamers $\left(k_r \to 0\right)$, and used to compute the nonlinear growth rate and stability boundary of each type of structure in various regimes. Ongoing work is focused on the problem of zonal flow saturation. In particular, we are attempting to compare the mechanisms of saturation by Kelvin-Helmholtz type instability and by feedback modulation of the drift wave spectrum.

Acknowledgements

We acknowledge stimulating and helpful discussions with X. Garbet, S. Benkadda, P. Beyer, L. Chen, F. L. Hinton, T. S. Hahm, Z. Lin, W. M. Nevins, M. N. Rosenbluth, V. Shapiro, P.L. Sulem and A. Smolyakov. This research was supported by U.S. D.O.E. Grant No. FG03-88ER53275.

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