Random Shearing by Zonal Flows and Transport Reduction

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Abstract

We assess the efficiency of random shearing by zonal flows in the reduction of scalar field transport. In the strong shear limit, we show that the flux scales with the RMS shear Ω_{rms} as $\propto \Omega_{rms}^{-1}$, similar to that of Ω^{-1} in the case of a coherent shear Ω. A random zonal flow with a finite correlation time τ_{ZF} renders the decorrelation of two nearby fluid elements less efficient, with a time scale $\tau_D = (\tau_\eta/\tau_{ZF}\Omega_{rms}^2)^{1/2}$ (τ_η is turbulent scattering (diffusion) time), leading to larger RMS scalar amplitude with a slightly different scaling of the flux ($\propto \tau_D/\Omega_{rms}$).

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Shear flows are ubiquitous in a variety of physical systems. Examples include differential rotation in galaxies and stars [1], zonal flows in major planets [2] and laboratory plasmas [3], equatorial winds and polar vortices in the earth atmosphere [4], shear layers in the oceans [5], etc. These large scale coherent structures play a distinctive role in determining transport in plasmas due to the dramatic effect of shearing on regulating turbulence. (see, e.g., [3]). This regulation is due to the fact that shearing generates fine spatial scales, as it advects different parts of an eddy at different rates, thereby leading to the distortion of turbulent eddies and their ultimate disruption when the dissipation becomes important on sufficiently small scales. The efficient generation of small-scales by shearing enhances the overall dissipation rate (so–called enhanced decorrelation). Remarkably, this process can even lead to the formation of transport barriers, as recently recognized in different systems [3–5].

The reduction of transport results from the change not only in the turbulence intensity but also in the phase shift (cross-phase). For instance, in the case of a passive scalar field χ [6], the transport is reduced as $\langle \chi v_x \rangle \propto \Omega^{-1}$ by a stationary linear shear flow $\mathbf{U} = x\Omega\hat{y}$, mainly because the turbulence amplitude decreases as $\langle \chi^2 \rangle \propto \Omega^{-5/3}$ with no significant change in the cross-phase ($\langle \chi v_x \rangle / \sqrt{\langle \chi^2 \rangle \langle v_x^2 \rangle} \propto \Omega^{-1/6}$) (cf. [7]). When the turbulent flow v_x evolves self-consistently, its amplitude is also reduced by shearing, resulting in a stronger reduction in the transport [8].

Shear flows, often encountered in a variety systems, are often self-generated by the underlying turbulence via Reynolds stress [9], and thus very likely to be structured and possibly even random in both space and time, on account of the broad range of their excitation via modulational instability. Thus, the flow pattern can be expected to have finite correlation time and complex spatial structure. These random shear flows, which are non-linearly driven by turbulence, are the so-called zonal flows. Zonal flows, for example, are shown to play a crucial role in regulating turbulence [10,11], and thus thought to trigger the formation of transport barriers (the so-called L-H transition) in fusion plasmas [12,13]. Therefore, it is important to understand how much transport is reduced, in general, by shear flows with finite correlation time τ_{ZF} [14] and complex spatial form. Nearly all of the previous work on shear flow regulation of transport has considered the mean shear case, only. In particular, we note that the exegis of the theoretical question of the relation between fluctuation levels and transport dates back to the early 60's, and that during the past 10 years, a community consensus as to both the ubiquity and importance of zonal flows in drift wave turbulence has arisen. Thus, an analysis of the relation between fluctuations and transport in the presence of zonal flows is both relevant and long overdue. Such an analysis is crucial to the long-term goal of relating fluctuations to transport, since direct measurement of turbulent fluxes in the core of relevant plasmas remains too difficult, and because turbulence in such plasmas will surely generate zonal flows, so that the fluctuations-to-transport relation *must* be constructed *for* an environment with zonal flows. The stochastic nature of zonal flow shearing suggests that the fluctuations-to-transport relationship must necessarily be statistical, in nature, as well. We shall see that this suggests many new questions, too.

The purpose of the paper is to study the effect of random shearing by zonal flows on turbulence regulation within a scalar field model. Intuitively, it is clear that shearing becomes ineffective as $\tau_{ZF} \to 0$, since then a shear flow has no time to act on an eddy [14]. The critical value of the correlation time of the zonal flows τ_{cr}, below which the shearing effect is reduced, is roughly $\tau_\Omega = \Omega_{rms}^{-1}$, where $\Omega_{rms} = \langle \Omega^2 \rangle^{1/2}$ is the RMS value of shear. For $\tau_{ZF} < \tau_\Omega$, the effective shearing rate becomes $\Omega_{eff} = \tau_{eff}^{-1} = \tau_{ZF}\Omega_{rms}^2$ $(< \Omega_{rms})$. It is well known that in the strong shear limit (i.e., $\tau_\Omega/\tau_\eta \to 0$), shearing enhances the decorrelation rate of two nearby fluid elements to $\tau_\Delta^{-1} = (Dk^2\Omega^2)^{-1/3} = (\tau_\eta \tau_\Omega^2)^{-1/3}$, above the value $\tau_\eta = (Dk^2)^{-1}$ determined by turbulent scattering alone [15]. Here, D is the effective diffusivity including the effect of nonlinear mixing, and $1/k$ is the characteristic scale of turbulence. Note that $\tau_\Delta \gg \tau_\Omega$ in the strong shear limit. When the shearing is random, this rate is also reduced to $\tau_D^{-1} = (Dk^2\tau_{ZF}\langle\Omega^2\rangle^2)^{-1/2} = (\tau_\eta \tau_{eff})^{-1/2}$ due to inefficient shearing (i.e., $\tau_\Delta < \tau_D$ for $\tau_{ZF} < \tau_\Delta$). As τ_{ZF} becomes large enough to satisfy $\tau_{ZF} \gg \tau_D$, the results for the limit of a steady shear flow are recovered, since zonal flows can then be considered to be steady, albeit with complex spatial structure. Another interesting consequence of random shearing with no net mean shear ($\langle \Omega \rangle = 0$) is the replacement of resonance between the flow and turbulence (where a local Doppler shifted frequency vanishes) by a smooth, probabilistic interaction kernel. We recall that resonance underlies irreversibility, yielding a non-trivial scaling of scalar field transport. The scaling of the flux, however, turns out to be similar to that for the case of a steady linear shear flow, with $\Omega \to \Omega_{rms}$. In comparison, the amplitude of the scalar field will be shown to be increased slightly, with a different scaling, because of longer effective decorrelation time ($\tau_D > \tau_\Delta$).

Let us consider the transport of scalar fields χ by random turbulent flow $\mathbf{v}$ and random zonal flow $\mathbf{U} = U(x,t)\hat{y}$ with $\langle \mathbf{v} \rangle = \langle U \rangle = 0$ in the two dimensional (2D) x and y plane:

$$[\partial_t + U(x,t)\partial_y]\chi' = -v_x \partial_x \chi_0 + D(\partial_{xx} + \partial_{yy})\chi' \,, \tag{1}$$

where χ' and χ_0 are fluctuating and mean parts of χ, and D is the effective diffusivity, including nonlinear interaction. The random turbulent flow is assumed to have characteristic frequency ω_k and correlation time $\tau_c = \gamma_k^{-1}$, while zonal flows have correlation time τ_{ZF}. Since a zonal flow $U(x,t)$ is random, the scalar field flux $\Gamma = \langle\langle \chi' v_x \rangle\rangle$ should be averaged over the ensemble of zonal flows, in addition to that of the turbulent flow $\mathbf{v}$. We shall use angular brackets $\langle\rangle$ to denote the average over either one of the two, and use double angular brackets $\langle\langle\rangle\rangle$ to denote the average over the two. The two interesting limits, which we shall focus on here, are (a) when a zonal flow is temporally random on time scales $\tau_{ZF} > \tau_c$, with a fixed linear profile $U(x,t) = x\Omega(t)$, and (b) when the zonal flow is steady, but spatially complex ($U(x,t) = U(x)$) [16]. The average over the ensemble of zonal flows can be envisioned as a *time* average for the *former* and as a *spatial* average for the *latter*.

We first examine the case when zonal flows have finite correlation time τ_{ZF} with a linear spatial profile, i.e., $U(x,t) = x\Omega(t)$. The degree to which randomness of zonal flows (finite τ_{ZF}) influences the dynamics depends on whether τ_{ZF} is smaller or larger than other characteristic time scales, such as the shearing time τ_Ω and decorrelation time τ_Δ. As we are interested in the strong shear limit, τ_Δ is taken to be much larger than τ_Ω throughout this paper. Thus, physically relevant cases are (i) $\tau_c < \tau_{ZF} < \tau_\Omega \ll \tau_D$ and (ii) $\tau_\Omega \ll \tau_c <$

$\tau_{ZF} \ll \tau_D$. Case (i) corresponds to delta-correlated turbulent (and zonal) flow, where the irreversibility mainly arises from the randomness of the flow while in case (ii), the zonal flow-wave resonance is the main source of irreversibility in the limit $\tau_{ZF} \to \infty$. It is illuminating that even without a complicated analysis, the scaling of flux in case (ii) can easily been obtained, since the long time average (over the ensemble of turbulence) of the flux does not depend on the dissipation, rendering it legitimate to take $\tau_{ZF} \to \infty$. Thus, we can simply take the result for a fixed shear flow $\langle \chi' v_x \rangle \propto \delta(\omega_k - x\Omega k_y)$, and then take its average over an ensemble of zonal flows. For simplicity, we perform the latter by assuming Gaussian probability for Ω as $dP[\Omega] = (1/\Omega_{rms}) d\Omega e^{-\Omega^2/2\Omega_{rms}^2}$:

$$\langle\langle \chi' v_x \rangle\rangle \propto \frac{1}{x k_y \Omega_{rms}} e^{-\omega_k^2/2x^2 k_y^2 \Omega_{rms}^2}. \tag{2}$$

Thus, a sharp resonance $\delta(\omega_k - x\Omega k_y)$ becomes a smooth, probabilistic interaction kernel, making the flux maximal for $\omega_k = \sqrt{2k_y^2 x^2 \Omega_{rms}^2}$, with its value $\propto \Omega_{rms}^{-1}$. Thus, the flux has a similar scaling with Ω_{rms} as that with Ω, in the case of a fixed shear flow. The same result (Eq. (2)) shall also be obtained through a more laborious calculation in what follows. It is important to note that the same analysis cannot be applied to $\langle\langle \chi'^2 \rangle\rangle$, since the long time average of $\langle \chi'^2 \rangle$ for a fixed shear is taken over the time longer than τ_D, which is much larger than τ_{ZF}. Thus, a more rigorous analysis is necessary.

To incorporate shearing effect in Eq. (2), we employ a time-dependent wavenumber $k_x(t)$ in the direction of the shear (i.e., shearing coordinate), by assuming

$$\chi'(\mathbf{x}, t) = \int d^2 k e^{i(k_x(t)x + k_y y)} \, \tilde{\chi}(k_x(t), k_y, t). \tag{3}$$

Upon using Eq. (3), and assuming $\partial_t k_x(t) = -k_y \Omega(t)$, Eq. (2) can easily be solved as

$$\tilde{\chi}(k_x(t), k_y, t) = -\partial_x \chi_0 \int d^2 k_1 \int^t dt_1 g(\mathbf{k}, t; \mathbf{k}_1, t_1) \, e^{-DQ(t_1)} \tilde{v}_x(k_x(t), k_y, t) \, . \tag{4}$$

Here, $Q(t_1) = k_y^2(t - t_1) + \int_{t_1}^t dt' k_x^2(t')$, and $g(\mathbf{k}, t; \mathbf{k}_1, t_1) = \delta(k_y - k_{1y})\delta(k_x - k_{1x} + k_{1y} \int_{t_1}^t dt_2 \Omega(t_2))$ is the Green's function for the evolution of χ'. From Eq. (4), the flux and mean square amplitude of χ', when averaged over the statistics of turbulent flow v_x, are as follows:

$$\langle \chi' v_x \rangle = -\frac{\partial_x \chi_0}{(2\pi)^2} \int^t dt_1 d^2k_1 e^{-ik_y x \int_{t_1}^t dt' \Omega(t') - DQ(t_1)} \phi(\mathbf{k}_1, t - t_1)\,, \tag{5}$$

$$\langle \chi'^2 \rangle = \frac{(\partial_x \chi_0)^2}{(2\pi)^2} \int^t dt_1 dt_2 d^2k_1 e^{-ik_y x \int_{t_1}^{t_2} dt' \Omega(t') - D[Q(t_1)+Q(t_2)]} \phi(\mathbf{k}_1, t_1 - t_2)\,. \tag{6}$$

Here, again $Q(t_i) = k_y^2(t - t_i) + \int_{t_i}^t dt' k_x^2(t')$ for $i = 1, 2$, and stationary and homogeneous turbulence for v_x has been assumed with $\langle \tilde{v}_x(\mathbf{k}_1, t_1)\tilde{v}_x(\mathbf{k}_2, t_2)\rangle = (2\pi)^2\delta(\mathbf{k}_1 + \mathbf{k}_2)\phi(\mathbf{k}_1, t_2 - t_1)$.

For case (i), we can take $\phi(\mathbf{k}, t_2 - t_1) = \tau_c \delta(t_1 - t_2)\psi(\mathbf{k})$ since τ_c is the shortest time scale in the system. Then, it is trivial to see that the effect of shearing for the flux vanishes as $\langle\langle \chi' v_x \rangle\rangle = \langle \chi' v_x \rangle = -(\tau_c \partial_x \chi_0/(2\pi)^2) \int d^2k \psi(\mathbf{k})$, which is consistent with the result for a steady shear flow. On the other hand, the effect of dissipation D, enhanced by zonal flow shearing, is critical to determining the amplitude $\langle\langle \chi'^2 \rangle\rangle$ because of the generation of fine scales in x (or large k_x), and requires the following quantity averaged over an ensemble of zonal flows:

$$I_z \equiv \langle e^{-Dk_y^2 \int_0^\tau dt' k_x^2(t')} \rangle\,. \tag{7}$$

Since the argument of the exponential is quadratic in Ω with non-vanishing mean value ($\partial_t k_x(t) = -k_y \Omega(t)$), the average can be evaluated for each term by assuming Gaussian statistics for Ω after expanding the exponential function. Of course, other forms of the probability distribution function should be considered, as well. For this average, shearing can be treated as a random walk over τ_D, since the former changes many times, so long as $\tau_{ZF} \ll \tau_D$. For instance, it is reasonable to take $\int_0^t dt_1 \int_0^t dt_2 \langle \Omega(t_1)\Omega(t_2)\rangle \simeq t\tau_{ZF}\langle \Omega^2 \rangle$. By using this and focusing on the strong shear limit $Dk_y^2 < \tau_{ZF}\langle \Omega^2\rangle$, we scale the time with $\tau_D = (Dk_y^2 \tau_{ZF}\langle \Omega^2 \rangle)^{-1/2}$ in the resulting average. This will reduce the time integral in Eq. (6) to $\int^t dt_1 e^{-DQ(t_1)} = \tau_D \int_0^\infty ds I_z(s) = \mathcal{C}\tau_D$ in the long time limit ($t \to \infty$), with a converging integral $C \equiv \int_0^\infty ds I_z(s)$. Thus, the amplitude becomes

$$\langle\langle \chi'^2 \rangle\rangle = \frac{\tau_c \tau_D \mathcal{C}}{(2\pi)^2} \int d^2k \phi(\mathbf{k}) \propto [Dk_y^2 \tau_{ZF} \langle \Omega^2 \rangle]^{-1/2}\,. \tag{8}$$

As compared with the amplitude $\langle \chi'^2 \rangle \propto \tau_\Delta$ ($> \tau_D$ for $\tau_{ZF} < \tau_D$) in the case of coherent shearing, random shearing clearly yields larger amplitude. Eq. (8) also explicitly shows that the amplitude increases, as τ_{ZF} decreases as it may be expected to. The upper bound on the amplitude is, however, given by $\tau_D = (Dk_y^2\langle\Omega^2\rangle)^{-1}$ in order to satisfy the assumption of the strong shear limit $Dk_y^2 < \tau_{ZF}\langle\Omega^2\rangle$. Note that for a steady flow with constant Ω, the argument of the exponential in Eq. (7) becomes proportional to τ^3, resulting in a characteristic time scale τ_Δ. When $\tau_{ZF} \gg \tau_D$, the zonal flow can be treated as steady, and thus the amplitude becomes proportional to τ_Δ. These results are summarized in Table 1.

For case (ii), we assume a Lorentzian frequency spectrum for ϕ as $\phi(\mathbf{k}, t_2 - t_1) = \int d\omega \psi(\mathbf{k}) e^{-i\omega(t_2-t_1)}\gamma_k/[(\omega-\omega_k)^2 + \gamma_k^2)]$. Then, after performing the ω integral, the ensemble average of the flux becomes

$$\langle\langle\chi' v_x\rangle\rangle = -\frac{\partial_x \chi_0}{(2\pi)^2}\int^t d\tau d^2k \psi(\mathbf{k})\langle e^{-ik_y x\int_0^\tau dt'\Omega(t')}\rangle\,, \tag{9}$$

where $\tau = t - t_1$. By using Gaussian statistics for Ω, the ensemble average over zonal flows can be approximated as $\langle e^{-ik_y x\int_0^\tau dt'\Omega(t')}\rangle = e^{-((k_y x)^2/2)\int_0^\tau dt' dt''\langle\Omega(t')\Omega(t'')\rangle} \simeq e^{-((k_y x)^2/2)\tau^2\langle\Omega^2\rangle}$. Note that zonal flow shearing was taken to be coherent over the time interval $t \in [0, \tau]$ since the effective shearing time τ_Ω is shorter than τ_{ZF} and also since turbulence varies on the fast time scale γ_k^{-1}, giving a major contribution to the τ integral from small τ. Thus, the remaining τ integral gives (by assuming $\omega_k > \gamma_k$)

$$\langle\langle\chi' v_x\rangle\rangle \simeq -\frac{\partial_x \chi_0}{2(2\pi)^{2/3}}\int d^2k \psi(\mathbf{k})\frac{e^{-\omega_k^2/(2k_y^2 x^2\langle\Omega^2\rangle}}{(k_y^2 x^2\langle\Omega^2\rangle)^{1/2}}\,. \tag{10}$$

It is amusing to see that the scaling in Eq. (10) is exactly what Eq. (2) predicted, which was obtained simply by taking an average of the existing result for a steady shear flow. We again note that the flux in Eq. (10) takes its maximum value for $\omega_k = \sqrt{2k_y x^2\langle\Omega^2\rangle}$, (which replaces the resonance condition), and is Ω^{-1}.

In order to compute the amplitude for case (ii), we note that the effect of dissipation enters on a long time scale (τ_D), compared to other time scales. Thus, we envision taking average of zonal flows over two different time scales T_1 and T_2, where $\tau_{ZF}, \tau_c < T_1 < \tau_D$ and $T_2 > \tau_D$. Thus, we can first ignore the dissipation (i.e., $D = 0$) and take average over T_1, and then take average over T_2 with $D \neq 0$. Then, similar analyses to those done previously, give us the following:

$$\langle\langle\chi'^2\rangle\rangle = \frac{(\partial_x\chi_0)^2}{(2\pi)^{2/3}}\int d^2k\psi(\mathbf{k})\frac{\tau_D \mathcal{C} e^{-\omega_k^2/(2k_y^2x^2\langle\Omega^2\rangle)}}{(k_y^2x^2\langle\Omega^2\rangle)^{1/2}} \propto \frac{1}{\langle\Omega^2\rangle(Dk_y^2\tau_{ZF})^{1/2}}\,, \tag{11}$$

where, again, $\mathcal{C} \equiv \int^\infty ds I_z(s)$ is a convergent integral. Eq. (11) reveals that the amplitude in the case of random shear is enhanced due to the inefficiency of turbulence regulation on account of the shear's random nature as compared to a steady shear, where $\langle\chi'^2\rangle \propto \tau_\Delta/\Omega$. For instance, the amplitude increases as τ_{ZF} becomes small, as expected. The upper limit on the amplitude is, however, again given by $\tau_D = (Dk_y^2\langle\Omega^2\rangle)^{-1}$. On the other hand, as τ_{ZF} becomes larger than τ_D, τ_D in Eq. (11) should be replaced by τ_Δ, since zonal flows can then be treated as steady flows.

We will now show that in the case of a steady zonal flow with complex spatial dependence, the results are similar to those in the case of linear shear flow, provided that Ω is replaced by the RMS shearing rate $\Omega_{rms} = \langle(\partial_x U)^2\rangle^{1/2}$, in agreement with [16]. Since it is plausible that the correlation length of zonal flows l_{ZF} can be comparable to that of turbulence l_c, we can no longer Fourier decompose χ' in x. Therefore, we Fourier transform only in y and introduce a phase function $g(x,t)$ as

$$\chi'(\mathbf{x},t) = \int dk_y e^{ik_y y + g(x,t)}\tilde{\chi}(k_y,x,t)\,. \tag{12}$$

By assuming that $|\partial_x g/g| \gg |\partial_x\tilde{\chi}/\tilde{\chi}|$, and by using the usual Fourier transform for v_x as $v_x(\mathbf{x},t) = \int dk_y e^{ik_y y}\overline{v}(k_y,x,t)$, Eq. (1) can easily be solved for $\tilde{\chi}$ as

$$\tilde{\chi}(x,k_y,t) = -\partial_x\chi_0\int^t dt_1 e^{-D[\overline{Q}(k,t)-\overline{Q}(k,t_1)]-g(x,t_1)}\overline{v}_x(k_y,x,t_1)\,. \tag{13}$$

Here, $\overline{Q}(k,t) = k_y^2 t + (k_y U')^2 t^3/3 + ik_y U'' t^2/2$ with $U' = \partial_x U$ and $U'' = \partial_{xx}U$. By exploiting 'steady and homogeneous turbulence' with the correlation function $\langle\overline{v}_x(x,k_y,t_1)\overline{v}_x(x,k_y',t_2)\rangle = (2\pi)\delta(k_y+k_y')\phi(k_y,t_2-t_1)$, we obtain:

$$\langle\chi' v_x\rangle = -\frac{\partial_x\chi_0}{(2\pi)}\int^t dt_1 dk_y e^{-ik_y U(t-t_1)-D[\overline{Q}(k,t)-\overline{Q}(k,t_1)]}\phi(\mathbf{k}_1,t-t_1)\,, \tag{14}$$

$$\langle\chi'^2\rangle = \frac{(\partial_x\chi_0)^2}{(2\pi)}\int^t dt_1 dt_2 dk_y e^{-ik_y U(t_2-t_1)-D[\overline{Q}(k,t)+\overline{Q}(-k,t)-\overline{Q}(k,t_1)-\overline{Q}(-k,t_2)]}\phi(\mathbf{k},t_2-t_1)\,. \tag{15}$$

First, it is easy to see that for a delta correlated flow v_x (i.e. $\tau_c \ll \tau_\Omega$), the flux is independent of U. However, the computation of $\langle\langle\chi'^2\rangle\rangle$ requires an average (over zonal flows) like $\langle e^{-2D(k_y U')^2(t^3-t_1^3)/3}\rangle$. Since the scaling of the amplitude with the RMS shear is of main interest, this can be done, by following a similar analysis done previously to obtain Eq. (8), and then by scaling the time with $(Dk_y^2\langle U'^2\rangle)^{1/3}$. The result is

$$\langle \chi'^2\rangle \propto \frac{(\partial_x \chi_0)^2\tau_c}{(2\pi)}\int dk_y \psi(k_y)\frac{1}{[Dk_y^2\langle U'^2\rangle]^{1/3}}\,. \tag{16}$$

Therefore, the scaling of the amplitude with RMS shear and D are the same as those in the case of a linear shear flow, provided that $\langle U'^2\rangle^{1/2}$ replaces constant Ω.

For $\tau_\Omega < \tau_c < \tau_D$, we again use Lorentzian frequency spectrum for v_x. After straightforward algebra using Gaussian statistics for U, U' and U'', and $\langle UU'\rangle = \langle U'U''\rangle = 0$, we can obtain the following scalings:

$$\langle \chi' v_x\rangle \simeq -\frac{\partial_x \chi_0}{(2\pi)}\int dk_y \psi(k_y)\frac{e^{-\omega_k^2/(2k_y^2\langle U^2\rangle)}}{|2k_y\langle U^2\rangle^{1/2}|}\,, \tag{17}$$

$$\langle \chi'^2\rangle \propto \frac{(\partial_x \chi_0)^2}{(2\pi)}\int dk_y \psi(k_y)\frac{e^{-\omega_k^2/(2k_y^2\langle U^2\rangle)}}{|2k_y\langle U^2\rangle^{1/2}|[Dk_y^2\langle \Omega^2\rangle]^{1/3}}\,. \tag{18}$$

Due to the spatial randomness of the zonal flow pattern, resonance between zonal flow and turbulence is smoothed out, with the maximum flux when $\omega_k = \sqrt{2k_y^2\langle U^2\rangle}$, similar to the temporally random case (see Eq. (10)). Therefore, the scalings of the amplitude with shear and D are basically the same as those in the case of a linear shear flow, provided that $\langle U'^2\rangle^{1/2}$ is replaced by Ω. We note that the curvature effect U'' does not appear in the final amplitude, since a strong shear limit $D^2k_y^2\langle U''^2\rangle/[Dk_y^2\langle U'^2\rangle]^{4/3} \sim ((Dk_y^2)^2/\langle U'^2\rangle)^{1/3}(l_c/l_{ZF})^2 \ll 1$ was assumed.

In summary, we have shown that the effect of random shearing of zonal flows on transport and fluctuation levels of scalar fields crucially depends on the zonal flow pattern and correlation time τ_{ZF}. For spatially random zonal flows $U(x,t) = U(x)$ ($l_{ZF} \gtrsim l_c$) with infinite memory time, the same scalings of flux and amplitude of scalar fields with $\langle U'^2\rangle^{1/2}$ are obtained as those with Ω in the case of a steady linear shear flow [16]. This is partly because the effect of curvature $\partial_{xx}U$ disappears in the strong shear limit, satisfying $((Dk_y^2)^2/\langle U'^2\rangle)^{1/3}(l_c/l_{ZF})^2 \ll 1$. More interesting results were found for zonal flows with finite correlation time τ_{ZF} (i.e., $U(x,t) = x\Omega(t)$) (see Table 1). For $\tau_c < \tau_\Omega < \tau_{ZF} \ll \tau_D$, the flux becomes independent of shearing to leading order, and major irreversibility arises from the randomness in the turbulent flow. In this case, $\langle\langle\chi'^2\rangle\rangle \propto \Omega_{rms}^{-1}$. In the physically more interesting case where $\tau_\Omega \ll \tau_c < \tau_{ZF} \ll \tau_D$, $\langle\langle\chi' v_x\rangle\rangle \propto \Omega_{rms}^{-1}$ while $\langle\langle\chi'^2\rangle\rangle \propto \Omega_{rms}^{-2} D^{-1/2}$. The scaling of the latter, which is different from $\langle\chi'^2\rangle \propto \Omega^{-5/3} D^{-1/3}$ in the case of coherent shearing Ω, is a result of a longer effective decorrelation time of fluid elements $\tau_D > \tau_\Delta$ induced by finite τ_{ZF}. As τ_{ZF} exceeds τ_D, zonal flows can be considered to be steady in time, thus recovering previous results.

Note that these results of this Letter raise at least as many questions as they answer. In particular, these results highlight the *great importance of the determination of both the frequency spectrum* (in particular, the correlation time τ_{ZF}) *and the probability distribution function (PDF) of zonal flows*, in both simulations and physical experiments. These are *not* merely academic questions, but are, indeed, crucial to determining the relation between fluctuations and transport. In particular, we have assumed a Gaussian PDF of zonal flows throughout this paper, but there are likely cases for which the PDF of zonal flows is exponential or even power law. Experimentally, a useful estimate on τ_{ZF} can be obtained from constructing the average two time correlation function of a zonal flow $V_E\hat{y}$, i.e. $\tau_{ZF} = \int_0^\infty dt \langle V_E(\tau) V_E(\tau+t)\rangle / \langle V_E(\tau)^2\rangle$, or from the width of the $m = 0$ frequency spectrum. Simulation results can guide the theoretical modeling and understanding of τ_{ZF}, and vice versus. An interesting recent simulation by Parker et al [17] has noted that kinetic trapped electron effects tend to increase the temporal stochasticity of the zonal flows, by shortening the correlation time. Our results would suggest then, that there would be a qualitative difference in the scaling of the cross phase notable in a comparison between the results of [17] and those of earlier, purely ion temperature gradient driven turbulence simulations, in which the zonal flow correlation times are much longer. We also note that it would be most interesting to compare the PDFs of zonal flow shears from these two types of simulations.

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TABLES

TABLE I. Summary of results for zonal flows with finite correlation time τ_{ZF}

	$\tau_c < \tau_{ZF} < \tau_\Omega \ll \tau_D$	$\tau_\Omega < \tau_c < \tau_{ZF} \ll \tau_D$	$\tau_D \ll \tau_{ZF}$
$\langle\langle\chi'^2\rangle\rangle$	Ω_{rms}^{0}	Ω_{rms}^{-1}	Ω_{rms}^{-1}
$\langle\langle\chi' v_x\rangle\rangle$	Ω_{rms}^{-1}	$\Omega_{rms}^{-2} D^{-1/2}$	$\Omega_{rms}^{-5/3} D^{-1/3}$