Electromagnetic secondary instabilities in electron temperature gradient turbulence

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Abstract

The electron temperature gradient (ETG) mode has been proposed as a primary mechanism for electron transport. The possibilities of magnetic secondary instabilities ("zonal" magnetic fields and magnetic "streamers") are investigated as novel potential mechanisms for electron transport regulation and enhancement, respectively. In particular, zonal magnetic field growth and transport regulation is investigated as an alternative to electrostatic zonal flows. Growth rates and implications for electron thermal transport are discussed for both electrostatic and magnetic saturation mechanisms. The possibility of magnetic streamers (meso-scale radial magnetic fields), and their potential impact on electron thermal transport, is also considered.

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I. INTRODUCTION

A key issue in magnetic confinement fusion is the understanding of microturbulence which is believed to drive anomalously high levels of transport. Although this problem has been intensively studied in the context of ion-temperature gradient (ITG) turbulence (likely the primary cause of ion particle and heat transport) a similar understanding of electron transport has not been achieved. There are several outstanding issues in the area of electron transport. The foremost issue is the need to identify the underlying instability process causing said transport. Several pieces of experimental evidence point towards the electron temperature gradient (ETG) mode. Work by Stallard *et. al.*[1] suggests that electron transport coefficents are weakly affected or unaffected by the shear flows believed to regulate the ITG modes, suggesting an electron transport mechanism which has a smaller characteristic scale and larger growth rate than the ion turbulence. The ETG mode satisfies both of these criteria. Their observations also suggest that the measured electron temperature gradient is close to the marginally stable value of the linear ETG mode. More recently, Ryter *et. al.* have published evidence that electron tranport and temperature profiles are determined by a critical gradient length[2]. However, it should be noted that there are other modes (notably short-wavelength collisionless trapped electron modes[3] and "drift-islands"[4,5]) which may also be able to explain some of these results. Indeed, it has not been conclusively shown that only one mode is responsible for electron transport. Also particularly challenging is to understand electron transport mechanisms which can function in the presence of transport barriers or other conditions which quench particle and thermal ion transport. In this paper, we restrict ourselves to the ETG mode.

Traditionally, one calculates the magnitude of turbulent transport based on mixing-length or quasilinear estimates of the turbulent flux. As such calculations require a determination of the turbulent spectrum, the question of nonlinear saturation mechanisms naturally arises. Analytic[6,7] and computational work[8] has demonstrated that ITG turbulence saturates via a nonlinear transfer of energy to shear flow modes, termed zonal flows, which are toroidally and poloidally symmetric. The zonal flows have a predominately poloidal flow component (certainly no radial flow), preventing them from tapping the free energy of the system to drive transport, and are damped due to ion-ion collisions. The combined system of zonal flows and turbulence can be described by a predator-prey type model, in which total wave energy is conserved. Because of the close analogy with the ITG mode, we investigate the ETG mode[9–11] for similar dynamics. It should be noted that such flows have been observed in simulations of ETG turbulence[12], in the special case of a magnetic field with a local minimum, and with $\lambda_{De} > \rho_e$. Due to intuitive expectations that electromagnetic effects are more important in the ETG case than ITG, we also investigate the possibility of zonal magnetic field generation as a possible saturation mechanism. Zonal magnetic fields are meso-scale poloidal magnetic fields with $k_y = k_z = 0$, which would saturate the turbulence via random magnetic shearing instead of "flow" shear (see Appendix B for more details). The generation of such fields in the context of explaining the low to high confinement (L-H) transition has been studied by Guzdar *et. al.*[13]. Zonal fields are also discussed in Gruzinov et. al.[14] and Diamond et. al.[7]. In this paper, we are interested in studying their general effectiveness as saturation mechanisms for ETG turbulence. We also consider zonal flow/field generation in the context of an random phase approximation (RPA) modulational instability, appropriate for *fully developed* wave-turbulence, rather than the four-mode coherent/parametric approach taken by Guzdar and co-workers.

A more recent developement in the study of ITG and ETG modes has been the discovery of "streamers",[7,11,15,16], which are radially extended convective cells. In particular, it has been argued that streamers represent a mechanism for describing the bursty or "intermittent" transport often observed in simulation and experiment, and provide a route to enhancing transport well beyond gyro-Bohm levels. In simulations of ETG turbulence, electrostatic streamers have been observed in certain parameter regimes[11]. Jenko *et. al.* argue that these streamers are a nessecity for enhancing electrostatic ETG transport to experimentally relevant levels. Following the zonal flow / field analogy, we also investigate the possibility of *magnetic streamers*. Magnetic streamers are *radial* mesoscale magnetic fields, produced by secondary instability, with the potential for greatly increased thermal transport. They are extended cells in B_x and B_y, providing a radial magnetic connection mechanism. They also represent an intriguing extension of a traditional convention of ETG turbulence, which is to heuristically invoke inverse cascade proceses as a mechanism for increasing the correlation length of the turbulence to the electron skin depth, with a resultant increase in turbulent flux.

The structure of the paper is as follows. In Section II, we present the analytic model used, and discuss the basic physics and linear dispersion relations. In Section III, zonal modes are investigated, whereas streamer physics is studied in Section IV. A summary and discussion of the results is given in Section V.

II. MODEL EQUATIONS

The full description of the electromagnetic ETG mode in general geometry, including nonlinear effects, requires a formidable set of equations. In this work, we use the model presented in Horton *et. al.*[10], in a local limit; a similar set of equations is used in Ref. 16. Equations for electron vorticity and pressure, as well as Ohm's law, are used to describe the evolution of the electrostatic potential, electron pressure, and parallel magnetic potential. This model assumes that there is no magnetic shear or parallel magnetic fluctuations, but does include the diamagnetic heat flux. It also assumes the ions to be fully adiabatic, since $k_\perp \rho_i \geq 1$. The perpendicular magnetic field fluctuations are then driven purely by the current arising from the fluctuating electron parallel velocity, allowing us to write $v_{||} = \nabla_\perp^2 A_{||}$, using the normalizations defined below. The dominant nonlinearities are assumed to be $\vec{v}_{E\times B} \cdot \vec{\nabla}_\perp f = \{\phi, f\}$ and $\tilde{\vec{B}}_\perp \cdot \vec{\nabla}_\perp f = -\frac{\beta}{2}\{A_{||}, f\}$, again using the normalizations defined below. The model equations are:

$$\left(-\tau + \nabla_\perp^2\right)\frac{\partial \phi}{\partial t} + \left(1 - \epsilon + (1+\eta)\nabla_\perp^2\right)\frac{\partial \phi}{\partial y} + \epsilon\frac{\partial p}{\partial y} + \nabla_\perp^2\frac{\partial A_{||}}{\partial z} = -\{\phi, \nabla_\perp^2 \phi\} + \frac{\beta}{2}\{A_{||}, \nabla_\perp^2 A_{||}\} \quad (1)$$

$$\left(\left(-\frac{\beta}{2} + \nabla_\perp^2\right)\frac{\partial}{\partial t} - \frac{\beta}{2}(1+\eta)\frac{\partial}{\partial y}\right) A_{||} - \frac{\partial}{\partial z}(\phi - p) = -\{\phi, \nabla_\perp^2 A_{||}\} - \frac{\beta}{2}\{A_{||}, \phi - p\} \quad (2)$$

$$\frac{\partial p}{\partial t} + (1 + \eta - \Gamma\epsilon(1-\tau))\frac{\partial \phi}{\partial y} + 2\Gamma\epsilon\frac{\partial p}{\partial y} + \Gamma\nabla_\perp^2\frac{\partial A_{||}}{\partial z} = -\{\phi, p\} + \Gamma\frac{\beta}{2}\{A_{||}, \nabla_\perp^2 A_{||}\} \quad (3)$$

The Poisson brackets are defined as $\{f, g\} = \hat{z} \cdot \left(\vec{\nabla} f \times \vec{\nabla} g\right)$. The various quantities are normalized as $\phi = (L_n/\rho_e)\,|e|\tilde{\phi}/T_e$, $A_{||} = (2L_n v_T/\beta\rho_e c)\,|e|\tilde{A}_{||}/T_e$, $p = (L_n/\rho_e)\,\tilde{p}/p_0$, $L_f = -d\ln f/dx$, $\eta = L_n/L_{T_e}$, $\epsilon = L_n/L_B$, $\tau = T_e/T_i$, $\beta = 8\pi p_0/B_0^2$, $x, y \to x, y\,/\,\rho_e$, $z \to z\,/\,L_N$, $t \to L_n t\,/\,v_{Te}$, and $\Gamma = 5/3$. In particular, we have normalized the field quantities to the mixing length level (i.e. ϕ, $A_{||}$, $p \simeq \rho_e/L_n$ according to mixing length estimates, or are ~ 1 with this normalization). Note that damping terms, particularly thermal conduction, are neglected here (restricting the validity of the equations to regiemes of weak collisionality, appropriate for the core region of tokamaks). Simulations by Labit and Ottaviani[17] suggests that their effects are weak.

Physically one can interpret the nonlinear terms as: electrostatic and magnetic Reynolds stresses driving the vorticity, current and magnetic field advection in the Ohm's law, and convection of pressure along with the magnetic Reynolds stress driving the pressure. It is also useful to bear in mind that Eqns. 1 and 2 suggest that the relevant basic length scale for ϕ is $\rho_{ei} = \rho_e/\tau^{1/2}$, while $A_{||}$ will scale with the collisionless skin depth $c/\omega_{pe} = \beta^{-1/2}$ (in the normalized units used here).

A. Basic Dynamics and Linear Physics

A number of authors have investigated and documented the linear physics of ETG modes (see, e.g. Ref. 9,10). Therefore, we provide only a brief overview here. First, it is useful to consider the energetics of the mode. Defining $\mathcal{E}_\phi = \frac{1}{2}\int d^3x \left(\tau|\phi|^2 + |\nabla_\perp\phi|^2\right)$, $\mathcal{E}_A = \frac{1}{2}\int d^3x \left(\frac{\beta}{2}|\nabla_\perp A_{||}|^2 + |\nabla_\perp^2 A_{||}|^2\right)$, and $\mathcal{E}_p = \frac{1}{2\Gamma}\int d^3x\ |p|^2$, it is easy to show that

$$\frac{\partial}{\partial t}\left(\mathcal{E}_\phi + \mathcal{E}_A + \mathcal{E}_p\right) = \left(\frac{1+\eta}{\Gamma} + \epsilon\tau\right)\int d^3x\ p\left(-\frac{\partial\phi}{\partial y}\right) \tag{4}$$

using the identity $\int d^3x f\{f,g\} = 0$. The energy of the system grows as the *electrostatic* turbulent flux $Q_{turb}^{ES} = \int d^3x\ pv_x = \int d^3x\ p\left(-\frac{\partial\phi}{\partial y}\right)$ extracts energy from the mean gradients. It is interesting to note that in this model, magnetic fluctuations resdistrbute energy between the fields, but that the electromagnetic flux $Q_{EM} = v_{||}B_x p/B_0$ does contribute not to the growth of total fluctuatuion energy.

Let us now consider the linear dispersion relation. Fourier analyzing in time and space, we can combine Eqns. 1 - 3 together to find the linear dispersion relation

$$\omega_2\left(\omega_1\omega_3 + \epsilon k_y\omega_4\right) - k_{||}^2 k_\perp^2\left(\Gamma\left(\omega_1 + \epsilon k_y\right) + \omega_3 - \omega_4\right) = 0 \tag{5}$$

where we have defined

$$\omega_1 = \left(\tau + k_\perp^2\right)\omega + \left(1 - \epsilon - (1+\eta)\,k_\perp^2\right) \tag{6}$$

$$\omega_2 = \frac{\beta}{2}\left(\omega - (1+\eta)\,k_y\right) + k_\perp^2\omega \tag{7}$$

$$\omega_3 = \omega - 2\Gamma\epsilon k_y \tag{8}$$

$$\omega_4 = 1 + \eta - \Gamma\epsilon\,(1-\tau)\,k_y \tag{9}$$

In the limit that $k_{||}$ is unimportant, and neglecting the diamagnetic heat flux, one can determine the dispersion relation to be

$$\left(\left(\frac{\beta}{2}+k_\perp^2\right)\omega-\frac{\beta}{2}\left(1+\eta\right)k_y\right)\left(\left(\tau+k_\perp^2\right)\omega^2+k_y\omega+\epsilon\left(1+\eta\right)k_y^2\right)\simeq 0 \tag{10}$$

The solutions are a marginally stable drift oscillation arising from the parallel dynamics, and the electrostatic curvature-driven ETG mode, which is the mode of interest. The solution in this limit is $\omega_0=\omega_r^0+i\gamma_0=-\frac{k_y}{2\left(\tau+k_\perp^2\right)}+i|k_y|\sqrt{\frac{\epsilon}{\tau+k_\perp^2}}\left(\eta-\eta_c\right)^{1/2}$, $\eta_c\simeq\frac{1}{4\epsilon\tau}-1$. It is important to note that as the model used is only valid for $k_\perp\rho_e\leq 1$, a more detailed derivation of quantities such as η_c to include full finite Larmor radius (FLR) effects is not nessecarily meaningful, and potentially misleading. We treat the finite β and $k_{||}$ effects perturbatively to find their contributions to the dispersion relation. One finds

$$\delta\omega_r = \frac{k_\perp^2k_{||}^2k_y}{2\left(\tau+k_\perp^2\right)}\frac{a\,d-b\,c}{\left(a\omega_r^0-bk_y\right)^2+a^2\gamma_0^2} \tag{11}$$

$$\delta\gamma = -\frac{1}{\gamma_0}\frac{k_\perp^2k_{||}^2}{2\left(\tau+k_\perp^2\right)}\frac{\left(a\omega_r^0-bk_y\right)\left(c\omega_r^0-dk_y\right)+ac\gamma_0^2}{\left(a\omega_r^0-bk_y\right)^2+a^2\gamma_0^2} \tag{12}$$

$a=\frac{\beta}{2}+k_\perp^2$, $b=\frac{\beta}{2}\left(1+\eta\right)$, $c=1+\Gamma\left(\tau+k_\perp^2\right)$, and $d=\eta+1-\Gamma$. It is easy to see that $k_{||}$ effects are stabilizing, and introduce a frequency shift whose sign is parameter dependant. The growth rate from Eqn. 5 is plotted in Fig. 1 for typical parameters ($\tau=1$, $\eta=3$, $\epsilon=0.1$, and $\beta=0.04$). The stabilizing effects of $k_{||}$ as well as FLR stabilization at high $k_\perp$ are clearly seen.

III. ZONAL MODE EQUATIONS

A. Zonal Mode Generation

We first consider zonal modes, because of greater familiarity with their nonlinear dynamics. Conceptually, we assume that there is a spectrum of non-axisymmetric, "fast" / small scale (small but finite $k_{||}$, $k_\perp\sim\rho_e^{-1}$) modes representing the turbulence. Then based on experience with the generic drift waves and the ITG mode[6,7] we average the base equations over fast time and space scales (denoted by tildes) and investigate the possibility of a modulational instability for a "slow" ($k\ll\rho_e^{-1}$) mode with poloidal and toroidal symmetry ($\partial/\partial y=\partial/\partial z=0$). For the slow mode, averaging yields:

$$\left(-\tau+\nabla_x^2\right)\frac{\partial\bar{\phi}}{\partial t} = -\overline{\{\tilde{\phi},\nabla_\perp^2\tilde{\phi}\}}+\frac{\beta}{2}\overline{\{\tilde{A}_{||},\nabla_\perp^2\tilde{A}_{||}\}}+\mathcal{C}_\phi\left(\bar{\phi}\right) \tag{13}$$

$$\left(-\frac{\beta}{2}+\nabla_x^2\right)\frac{\partial \bar{A}_\parallel}{\partial t} = -\overline{\{\tilde{\phi},\nabla_\perp^2\tilde{A}_\parallel\}}+\frac{\beta}{2}\overline{\{\tilde{A}_\parallel,\tilde{\phi}-\tilde{p}\}}+\mathcal{C}_A\left(\bar{A}_\parallel\right) \tag{14}$$

$$\frac{\partial \bar{p}}{\partial t} = -\overline{\{\tilde{\phi},\tilde{p}\}}+\Gamma\frac{\beta}{2}\overline{\{\tilde{A}_\parallel,\nabla_\perp^2\tilde{A}_\parallel\}}-\mathcal{C}_p\left(\bar{p}\right) \tag{15}$$

Note that the polodial symmetry ($\partial/\partial y = 0$) of the slow mode makes it completely insensitive to the *linear* drive terms of the base equations, and reflects that such modes are necessarily nonlinearly generated. The $\mathcal{C}_\phi$ term represents a generalization of the Rosenbluth-Hinton collisional damping term for electrostatic zonal flows[18], with $\mathcal{C}_A$ and $\mathcal{C}_p$ representing collisional parallel resistivity and diffusion for $\bar{A}_\parallel$ and $\bar{p}$, respectively. Physically, the zonal modes are damped by an effective friction between the kinetic electron response to the mode and trapped electrons. It should also be noted that in the spirit of analogy between ITG and ETG physics, one might expect $\mathcal{C}_\phi \propto \nu_{ee}$, and $\mathcal{C}_A \propto \nu_{ei}$, relative to ν_{ii} in the ITG case. As $\nu_{ee} \simeq \nu_{ei} \gg \nu_{ii}$, it is likely that collisional damping of zonal modes may be even more important in ETG than ITG turbulence.

We now assume that we can describe the underlying fluctuations via a quasi-linear approach, such that

$$\tilde{A}_{\parallel,k} \simeq k_z\frac{\omega_3-\omega_4}{\omega_2\omega_3-\Gamma k_\parallel^2 k_\perp^2}\tilde{\phi}_k = R_A\,\tilde{\phi}_k \tag{16}$$

$$\tilde{p}_k \simeq \frac{\omega_2\omega_4-\Gamma k_\parallel^2 k_\perp^2}{\omega_2\omega_3-\Gamma k_\parallel^2 k_\perp^2}\tilde{\phi}_k = R_p\,\tilde{\phi}_k \tag{17}$$

We can reexpress the nonlinear terms as functions of $|\tilde{\phi}_k|^2$, using these quasilinear responses, and known properties of the Poisson brackets (see Appendix A), as:

$$\left(\tau-\nabla_x^2\right)\frac{\partial\bar{\phi}}{\partial t} = -\nabla_x^2\int d^3k\left(1-\frac{\beta}{2}|R_A|^2\right)k_xk_y\,\delta|\tilde{\phi}_k|^2-\mathcal{C}_\phi\left(\bar{\phi}\right) \tag{18}$$

$$\left(\frac{\beta}{2}-\nabla_x^2\right)\frac{\partial\bar{A}_\parallel}{\partial t} = -i\nabla_x\int d^3k\left(k_\perp^2+\frac{\beta}{2}\left(1-R_p\right)-i\nabla_xk_x\right)k_yR_A\,\delta|\tilde{\phi}_k|^2-\mathcal{C}_A\left(\bar{A}\right) \tag{19}$$

$$\frac{\partial\bar{p}}{\partial t} = i\nabla_x\int d^3k\left(R_p+i\nabla_xk_x\Gamma\frac{\beta}{2}|R_A|^2\right)k_y\,\delta|\tilde{\phi}_k|^2-\mathcal{C}_p\left(\bar{p}\right) \tag{20}$$

To close these equations, we exploit the scale separation between the underlying turbulence and the slow mode by using the wave-kinetic equation to calculate the response of the turbulence to the zonal modes. Such an approach exploits the fact that large-scale modulations of the small-scale fields conserve the action or quanta number ($N_k = \mathcal{E}_k/\omega_k$, where $\mathcal{E}_k$ is the energy of mode k) of the small-scale fields. This approach is valid due to the time-scale separation between the slow and fast modes. Generically, there will be an adiabatic invariant of the form

$$N_k = N_k\left(|\tilde{\phi}_k|^2, |\tilde{A}_{||,k}|^2, |\tilde{p}_k|^2\right) \tag{21}$$

Standard substitution of the quasilinear relations allow us to write $N_k = N_k\left(|\tilde{\phi}_k|^2\right)$, which in turn allows us to express the modulated nonlinear drive terms as functions of N_k via $\delta|\tilde{\phi}_k|^2 = \left(\delta|\tilde{\phi}_k|^2/\delta N_k\right)\delta N_k = \Lambda_k\,\delta N_k$. The adiabatic invariant N_k couples the turbulence to the slow mode via the wave-kinetic equation:

$$\frac{\partial N_k}{\partial t} + \frac{\partial \omega_{nl}}{\partial \vec{k}}\cdot\frac{\partial N_k}{\partial \vec{x}} - \frac{\partial \omega_{nl}}{\partial \vec{x}}\cdot\frac{\partial N_k}{\partial \vec{k}} = \gamma_{nl}N_k - \Delta\omega N_k^2 \tag{22}$$

where ω_{nl} and γ_{nl} are the frequency and growth rate of the underlying turbulence in the presence of the slowly-varying fields, and the first term of the right hand side represents linear growth of the turbulence, while the second term corresponds to higher-order interactions. To find ω_{nl}, one can modify the linear mode equations to reflect that the primary effect of the slowly-varying mode on the small scales is convection of fast modes by the slow, via $\partial_t \to \partial_t + \{\bar{\phi},\}$, $k_{||} \to k_z - \frac{\beta}{2}\{\bar{A}_{||},\}$; we also note that inclusion of a slow varying pressure will create an effective pressure gradient $\eta_{eff} = \eta - \nabla_x\bar{p}$. One can then write ω_{nl} as the sum of the original linear dispersion relation and an effective Doppler shift from the slowly varying fields $\left(v_{g||} = \partial\omega_k/\partial k_z\right)$

$$\begin{aligned} \omega_{nl} &\simeq \omega\left(k_\perp, k_{||}\right) + \vec{k}_\perp\cdot\vec{\bar{V}}_{ExB} \\ &= \omega_k^{lin} + v_{g||}\delta k_{||} + \vec{k}_\perp\cdot\vec{\bar{V}}_{ExB} \\ &= \omega_k^{lin} - \vec{k}_\perp\cdot\left(\vec{\nabla}\left(\bar{\phi} - \frac{\beta}{2}v_{g||}\bar{A}_{||}\right)\times\hat{z}\right) \end{aligned} \tag{23}$$

where we have taken $\delta\omega_k/\delta\eta \simeq 0$, as it enters only through FLR effects. Eqn. 23 underscores that the small-scale turbulence will be sheared by *both* the electrostatic and electromagnetic potentials i.e. it feels both flow and magnetic shear (see Appendix B for a more complete discussion). It is also important to note that the growth rate is modified by the presence of of the slow modes. The modified growth rate can be expressed as

$$\gamma_{nl} \simeq \gamma_k + \frac{\beta}{2}\frac{\partial\gamma_k}{\partial k_z}\vec{k}\cdot\left(\vec{\nabla}\bar{A}_{||}\times\hat{z}\right) - \frac{\partial\gamma_k}{\partial\eta}\nabla_x\bar{p} \tag{24}$$

The wave-kinetic equation then takes the form

$$\begin{aligned} &\frac{\partial N}{\partial t} + \vec{v}_g\cdot\vec{\nabla}N + k_y\nabla_x^2\left(\bar{\phi} - \frac{\beta}{2}v_{g||}\bar{A}_{||}\right)\frac{\partial N}{\partial k_x} \simeq \\ &\left(\gamma_k - \frac{\beta}{2}\frac{\partial\gamma_k}{\partial k_z}k_y\nabla_x\bar{A}_{||} - \frac{\partial\gamma_k}{\partial\eta}\nabla_x\bar{p}\right)N - \Delta\omega N^2 \end{aligned} \tag{25}$$

Expressing the action density as the sum of a mean background and a coherent response ($N_k = \overline{N_k} + \delta N$), one can linearize the wave-kinetic equation to find an expression for δN

$$\delta N_q = -q^2 k_y \left(\bar{\phi}_q - \frac{\beta}{2} v_{g||} \bar{A}_q \right) R\left(q v_{gx}\right) \frac{\partial \overline{N_k}}{\partial k_x} \tag{26}$$
$$-iq \left(k_y \frac{\beta}{2} \frac{\partial \gamma_k}{\partial k_z} \bar{A}_q + \frac{\partial \gamma_k}{\partial \eta} \bar{p}_q \right) R\left(q v_{gx}\right) \overline{N_k}$$

γ_q and q are the growth rate and wavenumber of the large scale perturbation, and $R\left(q v_{gx}\right) = 1/\left(\Delta\omega_k + \left(\gamma_q + \imath q v_{gx}\right)\right)$, where $\Delta\omega_k$ now encompasses both the linear growth rate and nonlinear frequency shift of the underlying turbulence, and γ_q is the growth rate of the zonal modes.. One can now close the zonal mode equations via substitution of δN_q into Eqns. 13 - 15. However, it is useful to first consider the various k-space symmetries of the nonlinear drive terms and δN_q. In particular, examination of the quasilinear responses indicates that R_A is odd in k_z, while R_p is even. Addtionally, $v_{g||}$ and $\partial \gamma_k / \partial k_z$ are also odd in k_z, while $\partial \gamma_k / \partial \eta$ is even. Thus, upon substitution of δN_q into Eqns. 13 - 15, and integration over k_z, one finds that the equation for zonal magnetic potential decouples from the electrostatic potential and pressure equations. Therefore, *zonal magnetic field dynamics are effectively decoupled from electrostatic zonal flow dynamics*! One can also use k_x symmetry to show that $\bar{\phi}_q$ and $\bar{p}_q$ are essentially independant, with $\bar{p}_q$ acting as a passive scalar for the zonal mode case, as well as to simplify the the zonal field equation. The resulting equations of interest are:

$$\left(\tau + q^2\right) \frac{\partial \bar{\phi}_q}{\partial t} = q^4 \int d^3k \, k_y^2 \left(1 - \frac{\beta}{2} |R_A|^2\right) \frac{R\left(q v_{gx}\right)}{\Lambda_k} \left(-k_x \frac{\partial \bar{N}_k}{\partial k_x}\right) \bar{\phi}_q - \mathcal{C}_\phi\left(\bar{\phi}_q\right) \tag{27}$$

$$\left(\frac{\beta}{2} + q^2\right) \frac{\partial \bar{A}_q}{\partial t} = \frac{\beta}{2} q^4 \int d^3k \left(-k_y^2 R_A^{Re} v_{gz}\right) \frac{R\left(q v_{gx}\right)}{\Lambda_k} \left(-k_x \frac{\partial \bar{N}_k}{\partial k_x}\right) \bar{A}_q \tag{28}$$
$$+ \frac{\beta}{2} q^2 \int d^3k \left(k_\perp^2 + \frac{\beta}{2}\left(1 - R_p^{re}\right)\right) \left(k_y^2 R_A^{im} \frac{\partial \gamma}{\partial k_z}\right) \frac{R\left(q v_{gx}\right)}{\Lambda_k} \bar{N}_k \bar{A}_q - \mathcal{C}_A\left(\bar{A}_q\right)$$

It is now straightforward to find growth rates for the electrostatic and magnetic modes, so that

$$\gamma_q^\phi = \frac{q^4}{\tau + q^2} \int d^3k \, k_y^2 \left(1 - \frac{\beta}{2} |R_A|^2\right) \frac{R\left(q v_{gx}\right)}{\Lambda_k} \left(-k_x \frac{\partial \bar{N}_k}{\partial k_x}\right) - \nu_\phi \tag{29}$$

$$\gamma_q^A = \frac{\beta q^4/2}{\beta/2 + q^2} \int d^3k \left(-k_y^2 R_A^{Re} v_{gz}\right) \frac{R\left(q v_{gx}\right)}{\Lambda_k} \left(-k_x \frac{\partial \bar{N}_k}{\partial k_x}\right) \tag{30}$$
$$+ \frac{\beta q^2/2}{\beta/2 + q^2} \int d^3k \left(k_\perp^2 + \frac{\beta}{2}\left(1 - R_p^{re}\right)\right) \left(k_y^2 R_A^{im} \frac{\partial \gamma}{\partial k_z}\right) \frac{R\left(q v_{gx}\right)}{\Lambda_k} \bar{N}_k - \nu_A q^2$$

We have explicitly rewritten $\mathcal{C}_\phi$ and $\mathcal{C}_A$ to demonstrate their physical meanings (collisional friction and resistivity, respectively). Examination of Eqn. 29 shows that the growth rate of the electrostatic zonal flow is reduced relative to the ITG case because of fully adiabatic ions, and the stabilizing effects of the magnetic Reynolds stress. An interesting problem is to elucidate the conditions for if/when the magnetic Reynolds stress will completely stabilize the growth of $\bar{\phi}_q$. Examination of Eqn. 16 for R_A shows that $|R_A|^2 \propto k_z^2/k_y^2$, one can then write

$$k_y^2 \left(1 - \frac{\beta}{2} |R_A|^2\right) = k_y^2 - \frac{\beta}{2} k_z^2 f \left(k_\perp^2, \beta, \tau, \epsilon\right) \tag{31}$$

Eqn. 31 shows that the competition between the electrostatic and magnetic Reynolds stresses can be cast as the difference between mean k_y^2 and k_z^2 of the turbulence, or in other words, that $\left(\overline{k_y^2}\right)^{1/2}$ must be greater than a critical wavenumber k_c for ϕ to grow, where

$$k_c^2 = \frac{\beta}{2} \overline{k_z^2 f} \tag{32}$$

If one estaimates $\overline{k_z^2} \simeq (\epsilon/q)^2 \, \overline{k_y^2}$, it would appear that in general the magnetic Reynolds stress would be, at best, weakly stabilizing (that is, one would expect $\beta/2 \, (\epsilon/q)^2 f < 1$). One must quantify the "proportionality function" f, and especially its β dependence, to verify this suggestion.

An understanding of zonal magnetic field growth requires a more extensive analysis, as indicated by the relative complex of Eqn. 29 vs. 30. Such an analysis is most easily done by first considering some basic dependancies of the relevant quantities. In particular, one can estimate $R_A^{re} \propto -k_z/k_y$, $R_A^{im} \propto -\gamma_k k_z/k_y^2$, $\partial\gamma/\partial k_z \propto -k_z/\gamma_k$, $v_{gz} \propto sgn\,(ad - bc)\, k_z/k_y$. These expressions allow one to rewrite Eqn. 30 (momentarily ignoring the damping ν_A) as

$$\gamma_q^A \sim \frac{q^2\beta/2}{\beta/2 + q^2} \int d^3k \left(k_\perp^2 + L^{-2}\right) f_1 \frac{k_z^2}{\gamma_k} \frac{\bar{N}_k}{\Lambda_k} + sgn(ad - bc) \frac{q^4\beta/2}{\beta/2 + q^2} \int d^3k f_2 \frac{k_z^2}{\gamma_k} \frac{1}{\Lambda_k} \left(-k_x \frac{\partial \overline{N_k}}{\partial k_x}\right) \tag{33}$$

To interpret this result physically, it is useful to consider the the original Ohm's law equation (Eqn. 2). It is clear that the nonlinearites correspond to electrostatic convection of current and magnetic field fluctuations. Thus, the *electrostatic turbulence amplifies small-scale magnetic fluctuations into larger-scale magnetic fields!* Zonal magnetic field generation can be clearly be viewed as a kind of small-scale dynamo action[19]. Observing that this derivation has assumed $q \leq 1$, it is also interesting to note that the zonal field growth is driven primarily the term arising from modulation of the growth rate, rather than the frequency modulation term. In contrast, the modulation of the linear growth rate gives no contribution to the electrostatic mode growth rate. One can also note that like the linear fluctuations, the zonal electrostatic potential length scale is set by ρ_e, while the zonal field length scale depends on the collisionless skin depth. Finally, it should be noted that the zonal modes and turbulence form a connected system[6], and that the zonal modes back-react on the turbulence even as the turbulence generates these modes. For a more complete discussion of this issue, the reader is again refered to Appendix B.

B. Estimations of transport

As alluded to in the introductuion, the key question for any investigation such as this is "What level of transport is the mode expected to produce?" We address this question here. The turbulent radial heat flux Q_x is

$$Q_x = \langle \tilde{p}\tilde{v}_x \rangle = \langle \tilde{p}\left(-\frac{\partial \phi}{\partial y}\right)\rangle + \langle p v_{||} B_x \rangle = Q_x^{ES} + Q_x^{EM} \tag{34}$$

Keeping only second order correlations, the electromagnetic heat flux can be written as

$$\begin{aligned} Q_x^{EM} &= p_0 \langle \tilde{v}_{||}\tilde{B}_x \rangle + \bar{v}_{||}\langle \tilde{p}\tilde{B}_x \rangle + \bar{B}_x \langle \tilde{p}\tilde{v}_{||}\rangle \\ &= \frac{\beta}{2} p_0 \sum_k i k_y k_\perp^2 |A_k|^2 + \frac{\beta}{2}\bar{v}_{||}\sum_k i k_y R_p^* R_A |\phi_k|^2 + \bar{B}_x \sum_k k_\perp^2 R_A^* R_p |\phi_k|^2 \\ &= 0 \end{aligned} \tag{35}$$

The k-space symmetries of each term (k_y for the first, $k_{||}$ via R_A for the second and third) reduce the electromagnetic heat flux to zero. One might argue that the vanishing of the first term is a function of using triply periodic boundary conditions. This term can be rewritten as

$$\begin{aligned} \langle \tilde{v}_{||}\tilde{B}_x \rangle &= \frac{\beta}{2}\int dydz\, \nabla_\perp^2 A \frac{\partial A}{\partial y} \\ &= \frac{\beta}{2}\int dydz \nabla_x \left(\frac{\partial A}{\partial x}\frac{\partial A}{\partial y}\right) + \frac{\beta}{2}\frac{1}{2}\int dydz\, \nabla_y \left(\left(\frac{\partial A}{\partial y}\right)^2 - \left(\frac{\partial A}{\partial x}\right)^2\right) \\ &= \frac{\beta}{2}\nabla_x \int dydz \left(\frac{\partial A}{\partial x}\frac{\partial A}{\partial y}\right) \end{aligned} \tag{36}$$

Thus, the only remaining term upon averaging over flux surfaces is the radial gradient of the magnetic Reynolds stress, which will yield a transport much lower than that suggested by static stochastic field estimates[20]. With some consideration, this result should not be surprising, as it is well known that ambipolarity limits the particle diffusion predicted by such estimates. It should also be noted that consideration of the the energy equation (label) indicates that only electrostatic transport introduces energy into the system, while the magnetic nonlinearites redistribute the energy between various fields.

It is also instructive to consider potential transport arising from parallel conduction. In a collisional regieme (e.g. near the edge, but not in the core), a radial heat flux of the form $Q_x = \left(\tilde{B}_x/B_0\right) Q_{||} = -\kappa_{||}|\tilde{B}_x/B_0|^2 dT_0/dx$ might be expected, which would appear to have a potentially large magnitude. However, when one takes into account the fact that this expression is derived from $Q_{||} = -n_0\kappa_{||}\nabla_{||}T$, and $\vec{B}\cdot\vec{\nabla}T \simeq 0$, it becomes clear that collisional parallel transport along magnetic fluctuations cannot lead to experimentally relevant levels of electron heat transport (particularly in the core region).

We are then left with only the electrostatic heat flux:

$$Q_x^{ExB} = \langle \tilde{p}\tilde{v}_x \rangle \simeq p_{e0} v_{te} \sum_k (k_y\rho_e)\, Im\,(R_p)\, |\frac{e\tilde{\phi}_k}{T_e}|^2 \tag{37}$$

$$= 4\tau^2 (1+\eta)\, p_{e0} v_{Te} \sqrt{\frac{\epsilon}{\tau}}\, (\eta - \eta_c)^{1/2} \sum_k |k_y\rho_e|\, |\frac{e\tilde{\phi}_k}{T_e}|^2 \tag{38}$$

Using $Q = -n_0\chi d T_e/dx = p_{e0}\chi/L_T$, and defining $\chi_{GB} = \rho_e^2 v_{Te}/L_T$, one finds

$$\frac{\chi}{\chi_{GB}} = 4\tau^2 \frac{1+\eta}{\eta^2} \sqrt{\frac{\epsilon}{\tau}}\, (\eta-\eta_c)^{1/2} \sum_k |k_y\rho_e| \left(\frac{L_n}{\rho_e}\right)^2 |\frac{e\tilde{\phi}_k}{T_e}|^2 \tag{39}$$

We are now left with estimating the saturated intensity level of the turbulence, which is accomplished via use of a simple re-expression of the previously derived equations. Eqns. 29 - 30 are rewritten as

$$\frac{\partial \bar{\phi}_q}{\partial t} = \Sigma_\phi \mathcal{E} \bar{\phi}_q - \nu_\phi \bar{\phi}_q \tag{40}$$

$$\frac{\partial \bar{A}_q}{\partial t} = \Sigma_A \mathcal{E} \bar{A}_q - \nu_A q^2 \bar{A}_q \tag{41}$$

$\mathcal{E} = \overline{N_k}/\Lambda_k = (L_n/\rho_e)^2\, |e\tilde{\phi}/T_e|^2$ is the intensity of the small-scale turbulence, and we have written $\gamma_q^\phi = \Sigma_\phi \mathcal{E}$ and $\gamma_q^A = \Sigma_A \mathcal{E}$. Note that the gradient length used for normalizing the base equations is L_n, but as the mode is driven by the temperature gradient, it is more appropriate to use L_T when estimating mixing-length transport coefficents. In steady-state, the turbulence intensity is set by the balance between the nonlinear growth and linear damping of the zonal mode. The saturated intensity (in normalized units) and corresponding thermal diffusivity (in unnormalized units) for the case of electrostatic zonal flow saturation are

$$\mathcal{E}_\phi = \nu_\phi \frac{\tau + q^2}{q^4} \sqrt{\frac{\epsilon}{\tau}} \frac{(\eta - \eta_c)^{1/2}}{k_0 - k_c} \tag{42}$$

$$\frac{\chi_\phi}{\chi_{GB}} = 4\tau\epsilon \frac{1+\eta}{\eta} (\eta - \eta_c) \left(\frac{\nu_\phi L_T}{v_{Te}}\right) \frac{\tau + (q\rho_e)^2}{(q\rho_e)^4} \frac{k_0}{k_0 - k_c} \tag{43}$$

where k_0 is a mean k_y of the turbulence, and k_c was defined in Eqn. 32. For zonal field satuation, one finds

$$\mathcal{E}_A = \nu_A \left(1 + \frac{q^2}{\beta}\right) \frac{k_0^{-1}}{(k_0^2 + L^{-2}) f_1 + q^2 f_2} \sqrt{\frac{\eta - \eta_c}{\epsilon^3 \tau}} q_B^2 \tag{44}$$

$$\frac{\chi_A}{\chi_{GB}} = \frac{4\tau}{\epsilon} \frac{1+\eta}{\eta} (\eta - \eta_c) \left(\frac{\nu_A L_T}{v_{Te}}\right) \left(1 + (q\,(c/\omega_{pe}))^2\right) q_B^2 \frac{\rho_e^{-3}}{k_0 \left((k_0^2 + L^{-2}) f_1 + q^2 f_2\right)} \tag{45}$$

Here, $\overline{k_z^2}$ is again estimated as $(\epsilon/q_B)^2 k_0^2$, where q_B is the safety factor.

Consideration of χ_A offers an intriguing possibility. If $q\,(c/\omega_{pe}) > 1$, Eqn. 45 suggests that $\chi_A \propto (\eta - \eta_c)/\beta$. Such a scaling would be very appealing, as it offers the possibility of good agreement with experiment. In particular, one achives a β^{-1}-dependant scaling, without invoking increased correlation lengths of the turbulence, and considering only electrostatic transport. This result offers not only an interesting route to a neo-Alcator type scaling, but may also offer some insight into the results presented in Ref. 21, which describes ETG turbulence creating a χ_e due only to electrostatic transport, but which exhibits β^{-1} scaling. Labit and Ottaviani also observe decreasing transport with increasing β[17]. Clearly, then, the saturation mechanism for zonal field is crucial. We have taken here the simplest possibility, which is a purely collisional damping with no β dependence. If, however, zonal field growth is limited by a "tertiary" instability[22], one could easily imagine that the β scaling of χ_e could readily change. For a zonal magnetic field, such an instability might take the form of something akin to a microtearing mode, rather than the Kelvin-Helmholtz-like modes described in[22]. Thus, stability of zonal fields is an issue that demands further investigation. The requirement of zonal field scale smaller than skin depth for the β^{-1} scaling also highlights the importance of investigating the scales of the secondary instabilities. In the context of β dependencies of χ_e, one should also consider the role of the $k_0 - k_c$ term for the electrostatic case, which represents the competition between electrostatic and magnetic Reynolds stresses. It is clear that the magnetic Reynolds stress is a stablizing factor for the electrostatic zonal flow, and should be more effective for increasing β. Qualitatively, increased stabilization of the zonal flow with β leads to a higher saturated intensity level, and thus a higher transport level. However, a more quantitiative investigation is needed. It is also interesting to note that both modes give different ϵ scalings. Finally, it should be noted that the absolute magnitudes of the predicited thermal diffusivities may be constrained by their explicit dependence on collisionality, which has been assumed to be small in the analytic model used here.

C. Discussion

These simple considerations of transport suggest several interesting questions whose answers could shed more light on the physics of electron transport. First, the physics of collisionality and zonal mode saturation remains a key issue. In ITG turbulence, the competition of "teritary" instabilities, back-reaction on the turbulence, and collisional flow damping as secondary instability saturation mechanisms is an open issue. Investigation of analogous tertiary instabilities for ETG secondary modes is an obvious question, and such studies are underway. In particular, whether such tertiary instabilities will be able to compete with ν_ϕ, $\nu_A \sim \nu_{ee}$ is of particular interest. A more detailed investigation of ν_A and tertiary instabilites of the zonal field is particuarly warrented in light of the potential β scalings for χ_e our analysis suggests. One could also make a more pessimistic observation, and note that if the relevant collisional time scale for ETG is truly ν_{ee}, it is possibile that the damping may kill the zonal modes outright unless the turbulence reaches a much higher intensity level, relative to the ITG case. The different effects of β on electrostatic and magnetic modes are also interesting. An intriguing question to ask is if there is a critical β at which zonal fields become the dominant saturation mechanism, rather than zonal flows. The limitation of negligible magnetic flutter transport is counterintuitive to the "convential wisdom" in ETG turbulence, which has often qualitatively invoked flutter transport as the dominant transport mechanisim. However, negligible flutter transport is in agreement with the simulation results of Jenko *et. al.*[11,21], as well as Labit and Ottaviani[17]. It would interesting to determine what additional physics could be added to the model (if any), to break this constraint. Unless such physics is found, this limitation would appear to invalidate many of the earlier models. One would also like to quantify the effects of magnetic shear and geometry on the transport, as well as the impact of non-adiabatic ions. Finally, we suggest that many of the predictions and questions raised in this section could be addressed via exisitng codes, not the least of which would be to simply see if zonal fields are in fact generated in ETG simulations.

IV. MAGNETIC STREAMERS

In both ETG and ITG simulations, radially extended electrostatic convective cells are observed. These cells, termed streamers, are found to greatly enhance the turbulent transport. The possibility of zonal magnetic fields naturally leads to question of magnetic streamers. By magnetic streamers, we mean radially extended convective cells in B_x. They would be similar to magnetostatic convective cells, but are expected to have a finite real frequency. Based on previous analytic studies of electrostatic streamers in ITG turbulence, we undertake a similar study here. The approach used is similar to that of the zonal case, except now we look for modes with $\nabla_x \ll \nabla_y$, and $\nabla_z \simeq 0$. Structurally, these equations will be similar to those of the zonal modes, and in particular, it is clear that the k_z symmetry of the fluctuations will also decouple the magnetic streamer from the electrostic mode. The equation for magnetic streamer modes is

$$\left(\frac{\beta}{2} - \nabla_y^2\right)\frac{\partial \bar{A}_{\|}}{\partial t} + \frac{\beta}{2}(1+\eta)\frac{\partial \bar{A}_{\|}}{\partial y} = \overline{\{\tilde{\phi}, \nabla_\perp^2 \tilde{A}_{\|}\}} - \frac{\beta}{2}\overline{\{\tilde{A}_{\|}, \tilde{\phi} - \tilde{p}\}} + \nu_A \nabla_y^2 \bar{A}_{\|} \tag{46}$$

The equation is again closed via an appeal to wave-kinetics, with the adiabatic invariant response now

$$\delta N_q = \left(-q^2 k_x v_{g\|}\frac{\partial \overline{N_k}}{\partial k_y} + iqk_x\frac{\partial \gamma}{\partial k_z}\overline{N_k}\right)\frac{\beta}{2}R\left(qv_{gy}\right)\bar{A}_q \tag{47}$$

Carrying out the analysis is a similar fashion to the zonal mode, one finds that the streamer has a real frequency Ω_q and growth rate γ_q, which are

$$\Omega_q = \frac{\beta}{2}\frac{(1+\eta)\,q}{\beta/2+q^2} \tag{48}$$

$$\gamma_q = \frac{q^4\beta/2}{\beta/2+q^2}\int d^3k\,\left(-k_x^2 R_A^{Re} v_{gz}\right)\frac{R\left(qv_{gx}\right)}{\Lambda_k}\left(-k_y\frac{\partial \bar{N}_k}{\partial k_y}\right) + \frac{q^2\beta/2}{\beta/2+q^2}\int d^3k\,\left(k_\perp^2 + \frac{\beta}{2}\left(1-R_p^{re}\right)\right)\left(k_x^2 R_A^{im}\frac{\partial\gamma}{\partial k_z}\right)\frac{R\left(qv_{gx}\right)}{\Lambda_k}\bar{N}_k - \nu_A q^2 \tag{49}$$

It is useful to note that the structure of the streamer growth rate is quite similar to that of the zonal field growth rate, suggesting that $k_x - k_y$ asymmetries in the spectrum may be crucial for determining which has the larger growth rate.

Having established the potential for magnetic streamer growth, it is important to assess their importance via investigating the transport they are expected to produce. One is immediately confronted with the fact that in the model used, magnetic fluctuations cannot induce a flux (see Sec. III B). Several ways of extending the model which might allow significant flutter transport present themselves. The first is to appeal to additional physics which could alter the phase shift between $v_{||}$ and B_x. What such a mechanism would be (perhaps a current contribution from non-adiabatic ions), and whether the phase could be altered strongly enough to have a meaningful impact, are unclear. Alternatively, one might search for a way to overcome the objections of Sec. III B to transport due to parallel conduction along fluctuating field lines. However, to make such a claim, one should have a better understanding of the role of collisionality for large-scale modes. Perhaps the most appealing possibility is to relax the restriction on ∇_z, which would lead to *linear* couplings betwen the magnetic and electrostatic streamers. Indeed, simulations by Beyer *et. al.*[15] suggest that streamers are in fact composed of many different poloidal and toroidal mode numbers. In contrast to the zonal case, where the electrostatic and magnetic modes completely decouple, one would have a single "electromagnetic" streamer with both electrostatic and magnetic components; these components would have independent nonlinear driving terms, but coupled linear drives. Of particular interest would be to investiagte whether the linear stabilizing properties of the magnetic component (analogous to the line-bending stabilization effect of magnetic fluctuations on the linear mode) or its nonlinear drive are dominant when it couples to the electrostatic component. Such calculations are left for a future publication. However, one observes that many of the same limitations and issues rasied in the previous section appear again here, highlighting the need to quantify the role of collisional damping and tertiary instabilities (e.g. the physical mechanisms which determine streamer intensity) for streamers as well as zonal modes.

V. CONCLUSIONS

A thorough understanding of electron transport remains an open challenge to the magnetic fusion community. The ETG mode is often invoked as a potential mechanism for explaining the anomalously high electron transport. In this paper, we have investigated the possibility of secondary electrostatic and magnetic instabilities as potential saturation and transport regulation mechanisms. In particular, we have investigated in detail zonal magnetic fields as novel saturation mechanisms for the turbulence. Zonal magnetic fields are generated via electrostatic convection of magnetic field and current fluctuations, in clear analogy with mean-field dynamo theory, and saturate the turbulence via random magnetic shearing. It has been demonstrated that the underlying $k_{||}$ symmetry of the ETG mode leads to a decoupling of the zonal magnetic field from the "traditional" electrostatic zonal flow. We have also extended the idea of magnetic secondary instabilities to streamers. For streamers with $q_{||} \simeq 0$, one again has decoupled electrostatic and magnetic streamers, where as these modes will linearly couple into a single "electromagnetic" streamer for finite $q_{||}$. More detailed studies of streamer physics in ETG are currently underway.

Our investigations have raised as many questions as they have answered. The need for further study of magnetic Reynolds stress inhibition of zonal flow growth has been demonstrated. The inablity of magnetic flutter to induce transport seems to invalidate many of the more qualitative models of electron thermal transport, but appears to be a direct consequence of the relation between current and magnetic field. Most importantly, the need for a detailed understanding of the saturation mechanisms for ETG zonal modes and streamers, both electrostatic and magnetic, is a recurring consequence of our analysis. In particular, quantifying "tertiary" instabilities and collisional damping for the various secondary instabilities is key. It would also be useful to extend the analysis to a nonlocal model, which would introduce magnetic shear into the dynamics. Quantifying the role of magnetic shear in tertiary instabilites, particularly for the zonal field and magnetic streamer, would be of particular interest. Another crucial issue for both ITG and ETG turbulence is that of pattern selection- that is, whether zonal modes or streamers are preferentially generated. At this time, the issue is unresolved, but will be addressed in future publications. Finally, we note that many of these questions should be tractable to analysis by both existing simulations, and extensions of existing analytic ITG investigations.

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APPENDIX A: PROPERTIES OF POISSON BRACKETS

The Poisson brackets $\{f, g\} = \left(\vec{\nabla} f \times \vec{\nabla} g\right) \cdot \hat{z}$ offer a convient shorthand notation for writing many of the nonlinear terms encountered in plasma physics. In the course of modulation stability analysis, it is often helpful to rewrite the Poisson brackets in terms of partial derivatives acting on both f and g. In particular, the following identities are often found to be of use:

$$\{f, g\} = \nabla_y \left(\frac{\partial f}{\partial x} g\right) - \nabla_x \left(\frac{\partial f}{\partial y} g\right) \tag{A1}$$

$$\{f, \nabla_\perp^2 f\} = \left(\nabla_y^2 - \nabla_x^2\right) \left(\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\right) - \nabla_x \nabla_y \left(\left(\frac{\partial f}{\partial y}\right)^2 - \left(\frac{\partial f}{\partial x}\right)^2\right) \tag{A2}$$

$$\begin{aligned} \{f, \nabla_\perp^2 g\} &= \nabla_y^2 \left(\frac{\partial f}{\partial x}\frac{\partial g}{\partial y}\right) - \nabla_x^2 \left(\frac{\partial f}{\partial y}\frac{\partial g}{\partial x}\right) - \nabla_x \nabla_y \left(\left(\frac{\partial f}{\partial y}\frac{\partial g}{\partial y}\right) - \left(\frac{\partial f}{\partial x}\frac{\partial g}{\partial x}\right)\right) && \text{(A3)} \\ &- \{\frac{\partial f}{\partial x}, \frac{\partial g}{\partial x}\} - \{\frac{\partial f}{\partial y}, \frac{\partial g}{\partial y}\} \\ &= \nabla_y^2 \left(\frac{\partial f}{\partial x}\frac{\partial g}{\partial y}\right) - \nabla_x^2 \left(\frac{\partial f}{\partial y}\frac{\partial g}{\partial x}\right) - \nabla_x \nabla_y \left(\left(\frac{\partial f}{\partial y}\frac{\partial g}{\partial y}\right) - \left(\frac{\partial f}{\partial x}\frac{\partial g}{\partial x}\right)\right) && \text{(A4)} \\ &+ \nabla_x \left(\frac{\partial^2 f}{\partial x \partial y}\frac{\partial g}{\partial x} + \frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial y^2}\frac{\partial g}{\partial y}\right) - \nabla_y \left(\frac{\partial^2 f}{\partial x^2}\frac{\partial g}{\partial x} + \frac{\partial^2 f}{\partial x \partial y}\frac{\partial g}{\partial y}\right) \end{aligned}$$

APPENDIX B: GENERALIZED EFFECT OF RANDOM SHEAR AND GROWTH RATE MODULATION ON SMALL-SCALE TURBULENCE

It has been noted previously[6,7] that the coupled system for electrostatic zonal flows and drift waves form a closed system (of a predator-prey form) which conserves total energy. While the zonal flows are generated by the drift waves, they also back-react on the turbulence via random shearing in k-space. The back-reaction can easily be computed via quasi-linear formalism, and one finds coupled equations of the form $(\bar{N}_k = \langle N_k \rangle)$

$$\frac{\partial \phi_q^{ZF}}{\partial t} = q^2 \int_{k>k_0} d^3k\, k_y^2 \frac{R\left(q v_{gx}\right)}{\Lambda_k} \left(-k_x \frac{\partial \langle N_k \rangle}{\partial k_x}\right) \tag{B1}$$

$$\frac{\partial\langle N_k\rangle}{\partial t} = \frac{\partial}{\partial k_x}\left(q^4 k_y^2 R\left(q v_{gx}\right)|\phi_q^{ZF}|^2\right)\frac{\partial\langle N_k\rangle}{\partial kx} + \gamma\langle N_k\rangle - \Delta\omega\langle N_k\rangle^2 \tag{B2}$$

It is easy to see that Eqn. B2 describes how the random flow shear leads to diffusion of the turbulence in k_x, and will thus increase $\langle k_x^2\rangle$ (i.e. "eddy shearing"). We now wish to extend this result to include both the random magnetic shearing effects of the zonal field, as well as the effects of modulating the growth rate by the zonal magnetic field and zonal pressure. The theory of random shearing by both zonal fields and flows is developed in Sec. III A. As usual, ray chaos - namely, the overlap of wave group and zonal phase velocitiy resonances, is necessary for the applicability of quasi-linear theory. For a unified treatment of all effects, we rewrite the equation for $\langle N_k\rangle$ as

$$\begin{aligned}\frac{\partial\langle N_k\rangle}{\partial t} &= \frac{\partial}{\partial k_x}\left(q^2 k_y\left(\phi_{-q} - \frac{\beta}{2}v_{g||}A_{-q}\right)\delta N_q\right) - iq\left(\frac{\beta}{2}\frac{\partial\gamma_k}{\partial k_z}k_y A_{-q} - \frac{\partial\gamma_k}{\partial\eta}p_{-q}\right)\delta N_q \\ &+ \gamma_k\langle N_k\rangle - \Delta\omega\langle N_k\rangle^2\end{aligned} \tag{B3}$$

Eqn. 26 for δN_q can the be substituted in, giving the generalized description for the back-reaction:

$$\begin{aligned}\frac{\partial\langle N_k\rangle}{\partial t} &= \frac{\partial}{\partial k_x}\left(q^4 k_y^2 R\left(q v_{gx}\right)\left|\phi_q - \frac{\beta}{2}v_{g||}A_q\right|^2\right)\frac{\partial\langle N_k\rangle}{\partial k_x} \\ &+ q^2 R\left(q v_{gx}\right)\left|\frac{\beta}{2}\frac{\partial\gamma_k}{\partial k_z}k_y A_q - \frac{\partial\gamma_k}{\partial\eta}p_q\right|^2\langle N_k\rangle + \gamma_k\langle N_k\rangle - \Delta\omega\langle N_k\rangle^2\end{aligned} \tag{B4}$$

$$\Rightarrow \frac{\partial\langle N_k\rangle}{\partial t} = \frac{\partial}{\partial k_x}D_{EM}\frac{\partial\langle N_k\rangle}{\partial k_x} + \gamma_{NL}\langle N_k\rangle + \gamma_k\langle N_k\rangle - \Delta\omega\langle N_k\rangle^2 \tag{B5}$$

Thus, including zonal magnetic fields and pressure leads to a k_x diffusion coeffecient that reflects the electromagnetic character of the shearing, as well as a quasilinear growth rate via modulation of the linear growth rate. The new diffusion coefficent is an intutitve generalization of the electrostatic case, with $\phi \to \phi - \frac{\beta}{2}v_{g||}A$. For the case of zonal magnetic field generation, it is useful to note that as both β and $v_{g||}$ are small quantities, it is possibile that random magnetic shearing may not be as effective a saturation mechanism as the flow shear. Clearly, case-by-case quantitative analysis is required.

It is also interesting to consider the nonlinear growth rate created via modulation of the linear growth rate, and in particular, the effects of the zonal pressure. As $\partial\gamma_k/\partial\eta \sim (\eta-\eta_c)^{-1/2}$, there is the possibility of the zonal pressure introducing significant energy into the turbulence. A nonlinear modulation analysis, then, is required to treat the regieme near marginality ($\eta \sim \eta_c$). It was found that for zonal modes, the pressure is essentially generated as a passive scalar by the electrostatic mode, and uncorrelated with the zonal magnetic field. Thus, in the electrostatic case, one might expect a competition between the random shearing of the zonal flow as a saturation mechanism, and energy reintroduced into the turbulence via the zonal pressure. Again, this is an issue that should be addressed in a more quantitative fashion. Finally, we note that the introduction of zonal magnetic fields and pressures suggests interesting extensions of the predator-prey model developed in Diamond *et. al.*[6].

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Figure Captions

Fig. 1. Linear growth rate for $\tau = 1$, $\eta = 3$, $\epsilon = 0.1$, and $\beta = 0.04$.