

# Biophysics of Neurons and Networks

## Homework 1, Fall 2009

Due Tuesday October 13th  
Show all code/figures/work

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### Diffusion problems

1) Solving the diffusion equation

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$$

for a pulse of particles at  $x=0$  at  $t=0$ , where  $c(x,t)$  is concentration and  $D$  is the diffusion coefficient ( $m^2/s$ ), yields the impulse response (Green's function) for the diffusion equation

$$G(x,t) = \frac{1}{\sqrt{4\pi Dt}} e^{-x^2/4Dt}$$

Plot the concentration as a function of  $x$  and  $t$  using Matlab. Also, you can use the Green's function to derive the time evolution of diffusion for any arbitrary initial conditions  $c(x,0)$ . The way you do this is by convolving\*\*  $G(x,t)$  with  $c(x,0)$ :

$$c(x,t) = \int_{-\infty}^{\infty} c(\xi,0)G(x-\xi,t)d\xi$$

Do this for the initial conditions:  $c(x,0)=0$  for  $x<0$ ,  $c(x,0)=C_0$  for  $x>0$ , and plot what you get.

You can write the answer in terms of the error function if you like.

2) The impulse response in (1) is also a probability density function for the position of a single particle starting at  $x=0$  at  $t=0$ . Express the standard deviation in  $x$  as a function of time  $t$ .

This problem may be solved in two ways.

$$\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\langle x^2 \rangle} = \sqrt{\int_{-\infty}^{\infty} x^2 p(x) dx}$$

3) Matlab: Simulate a population of random walkers that start at  $x=0$  and move 1 step either to the left or right at each time step. Plot the standard deviation of the positions of your population as a function of the number of time steps elapsed ( $n$ ). Assuming that  $\sigma = n^\beta$  what is the value of beta?

4) Glutamate diffuses ( $D = 5 \times 10^{-6} \frac{cm^2}{s}$ ) across the synaptic cleft ( $\sim 20nm$ ) in order to bind to receptors on the postsynaptic membrane. Using the 1-dimensional impulse response, how long will it take for 50% of the particles to reach the postsynaptic membrane.

5) We are considering diffusion of a substance  $m$  down an axon of length  $L$  cm, where  $m$  is being consumed by some reaction at a constant rate  $\alpha$  (mol/s per unit length) all along the axon. The continuity equation is:

$$\frac{\partial c_m}{\partial t} = -\frac{\partial \phi_m}{\partial x} - \frac{\alpha_m}{A}$$

(the continuity equation for diffusion is just the same without the  $\alpha$  term. Do you understand why it is true?)

where  $A$  is the constant cross sectional area of the process, and  $\phi_m$  is the flux of  $m$  in moles/s.

a: Combine the continuity equation with Fick's 1st law:  $\phi = -D \frac{\partial c}{\partial x}$  to derive a modified form of the diffusion equation.

b: Show that in the steady state ( $\frac{\partial c_m}{\partial t} = \frac{\partial \phi_m}{\partial t} = 0$ ),

$c_m(x) = \frac{\alpha_m}{2DA}x^2 + a_0x + b_0$  is a solution and find values of the constants  $a_0$  and  $b_0$  corresponding to the boundary conditions  $c_m(0) = C_0$  and  $\phi_m(l) = 0$ .

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### Electrodiffusion / Nernst Problems

1) The Nernst-Planck equation relates the current density  $J$  to the flux due to diffusion  $\phi_{diffusion} = -D \frac{\partial c}{\partial x}$

to the flux due to drift of particles in an electric field

$$\phi_{drift} = -uzFc_m(x,t) \frac{\partial \psi}{\partial x},$$

$u$  is the mobility (velocity given a force) of the ion

$z$  is the valence of the ion

$F$  is Faraday's constant

$\psi$  is the electrical potential

With a little algebra, we get that the current density ( $A/cm^2$ ) is

$$J(x,t) = -uzFc(x,t) \frac{\partial}{\partial x} (RT \ln c(x,t) + zF\psi(x,t))$$

Show that at steady state ( $J=0$ , nothing is a function of time), we get the familiar

$$\psi(x) - \psi(x_0) = \frac{RT}{zF} \ln \frac{c(x_0)}{c(x)}$$

2) In 1942, Curtis and Cole studied how the membrane potential varies with the extracellular concentration of potassium (sign of the membrane potential is reversed in the figure below).

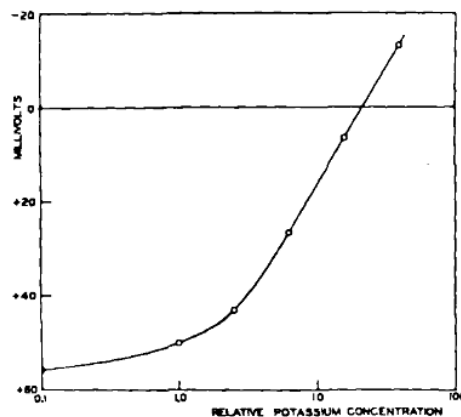


Fig. 2 Resting potential in millivolts vs. potassium concentration of the surrounding fluid. The concentration scale is in multiples of the potassium concentration of sea-water, 13 millimolar, and is logarithmic. At high potassium concentrations the curve is a straight line, the slope of which is nearly that of the potassium electrode. In the physiological range of concentrations the potential is nearly independent of the concentration.

Using the Goldman-Hodgkin-Katz voltage equation

$$V_{rest} = \frac{RT}{F} \ln \frac{P_K[K^+]_{out} + P_{Na}[Na^+]_{out} + P_{Cl}[Cl^-]_{in}}{P_K[K^+]_{in} + P_{Na}[Na^+]_{in} + P_{Cl}[Cl^-]_{out}}$$

Explain the observation that the relationship between extracellular  $K^+$  concentration and resting membrane potential becomes increasingly linear at high extracellular  $[K^+]$ . What should the slope of this relationship be at high extracellular potassium concentrations (in mV/10-fold increase in extracellular  $[K^+]$ )? Why might it be different in an experimental context?

3) How much does the sodium concentration change during an action potential?

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\* Contact if you are unfamiliar with Matlab, math used here, etc. We can setup a tutorial or something if enough people have similar concerns.

\*\* Unfamiliar with convolution? <http://www.jhu.edu/signals/convolve/> is intuitive. You should also know that this method of convolving the impulse response with initial conditions works for linear partial differential equations in general.