



The 11th International Conference on  
Materials & Mechanisms of Superconductivity (M2S)  
CICG, Geneva, Switzerland  
23 - 28 August 2015

# Tu-S17-KN1 / Alternative London electrodynamics, hole superconductivity, and the origin of the Meissner effect

Jorge E. Hirsch

*ucsd, United States of America*

## Abstract

An electrodynamic theory of superconductors that allows for the presence of electrostatic fields in their interior was proposed initially by the London brothers in 1933 [1] but discarded shortly thereafter in favor of the one generally accepted to this date. I will argue that the original theory is closer to the truth. The theory of hole superconductivity [2] predicts that superconductors expel negative charge from their interior to the surface resulting in an outward-pointing electric field in the interior of superconductors and a spin current near the surface. The superconductor is a giant atom, with macroscopically inhomogeneous charge distribution and macroscopic zero-point motion. The electrostatic energy cost is paid by lowering of quantum kinetic energy. The microscopic Hamiltonian is a dynamic Hubbard model [3] describing the expansion of atomic orbitals upon double electronic occupancy. Electrodynamic equations in the charge [4] and spin sectors [5] and resulting predictions that can be tested experimentally will be discussed. It is argued that the theory is consistent with existing experiments, provides a unified explanation for high and low temperature superconductivity [6,7], and indicates that high temperature superconductivity results from holes conducting through closely spaced negatively charged anions [8]. Unlike the conventional theory, it provides a dynamical explanation of the Meissner effect [9,10].

## References

- [1] F. London and H. London, The Electromagnetic Equations of the Supraconductor, Proc. Roy. Soc. A 149, 71 (1935).
- [2] References in <http://physics.ucsd.edu/~jorge/hole.html>.
- [3] J.E. Hirsch, Dynamic Hubbard model for solids with hydrogen-like atoms, Phys. Rev. B 90, 104501 (2014) and references therein.
- [4] J.E. Hirsch, Electrodynamics of superconductors, Phys. Rev. B 69, 214515 (2004).
- [5] J.E. Hirsch, Electrodynamics of spin currents in superconductors, Ann. Phys. (Berlin) 17, 380 (2008).
- [6] J.E. Hirsch, Materials and mechanisms of hole superconductivity, Physica C 472, 78 (2012).
- [7] J.E. Hirsch and F. Marsiglio, Hole superconductivity in  $\text{SH}_{2\text{S}}$  and other sulfides under high pressure, <http://arxiv.org/abs/1412.6251> (2014).
- [8] J.E. Hirsch, Hole Superconductivity, Phys. Lett. A134, 451 (1989).
- [9] J.E. Hirsch, Dynamic Hubbard model: kinetic energy driven charge expulsion, charge inhomogeneity, hole superconductivity, and Meissner effect, Physica Scripta 88, 035704 (2013).
- [10] J.E. Hirsch, The origin of the Meissner effect in new and old superconductors, Physica Scripta 85, 035704 (2012).

# *Alternative London electrodynamics, hole superconductivity, and the origin of the Meissner effect*

**J.E. Hirsch, UCSD**

**M<sup>2</sup>S 2015**

**An electrodynamic theory of superconductors that allows for the presence of electrostatic fields in their interior was proposed initially by the London brothers in 1933 but discarded shortly thereafter in favor of the one generally accepted to this date. I will argue that the original theory is closer to the truth. The theory of hole superconductivity predicts that superconductors expel negative charge from their interior to the surface resulting in an outward-pointing electric field in the interior of superconductors and a spin current near the surface. The superconductor is a giant atom, with macroscopically inhomogeneous charge distribution and macroscopic zero-point motion. The electrostatic energy cost is paid by lowering of quantum kinetic energy. The microscopic Hamiltonian is a “dynamic Hubbard model” describing the expansion of atomic orbitals upon double electronic occupancy. Electrodynamic equations in the charge and spin sectors and resulting predictions that can be tested experimentally will be discussed. It is argued that the theory is consistent with existing experiments, provides a unified explanation for high and low temperature superconductivity, and indicates that high temperature superconductivity results from holes conducting through closely spaced negatively charged anions. Unlike the conventional theory, it provides a dynamical explanation of the Meissner effect.**

**References in: <http://physics.ucsd.edu/~jorge/hole.html>**

# Superconducting materials

**‘Conventional’ superconductors** : superconducting elements, thousands of alloys and compounds.  $T_c^{\max} \sim 23\text{K}$  (old days)

- described by London’s electrodynamic theory (1935)
- **described by BCS theory (1957): electron-phonon, s-wave**  
MgB<sub>2</sub> (2001) ( $T_c=39\text{K}$ ), H<sub>2</sub>S? (200K)

**‘Unconventional’ superconductors**: high T<sub>c</sub> cuprates (1986), heavy fermion (1979), organic (1979), Sr<sub>2</sub>RuO<sub>4</sub> (1994), Fe-As, FeSe compounds (2008)...

$T_c \sim .1\text{K to } 150\text{K}$

- described by London theory
- **NOT described by BCS theory : no electron-phonon, no-s-wave**

**‘Undetermined’ superconductors** (conventional or maybe not?):

Bismuthates (1975) (34K), C<sub>60</sub> (1991) (33K), borocarbides (1993) (23K), BiS<sub>2</sub> (2012) (10K),...

# Superconducting materials

# PHYSICA



Recognized by the European Physical Society

# C

Volume 514, 15 July 2015

p 1-444

## **SUPERCONDUCTIVITY AND ITS APPLICATIONS**

Special Issue

Superconducting Materials:  
Conventional, Unconventional  
and Undetermined

Dedicated to Theodore H. Geballe on the year  
of his 95th birthday

Editors:

J.E. Hirsch  
M.B. Maple  
F. Marsiglio

**96 authors**

**32 classes of materials:**

**12 conventional**

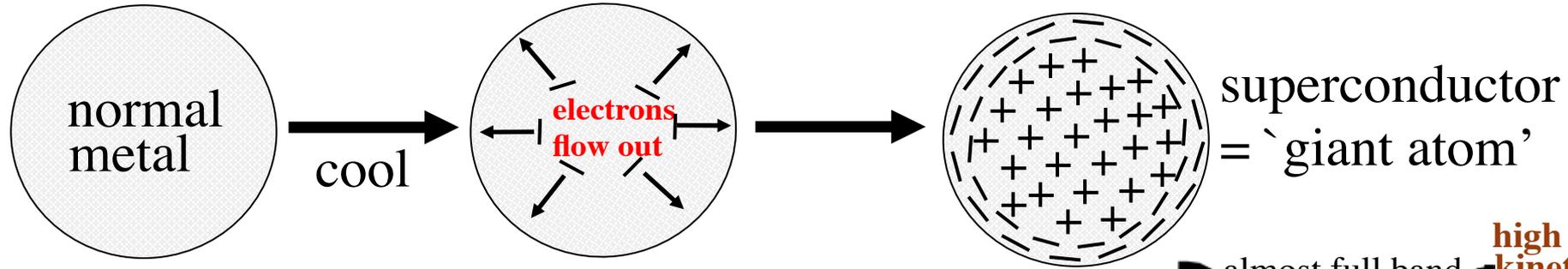
**11 unconventional**

**9 undetermined**

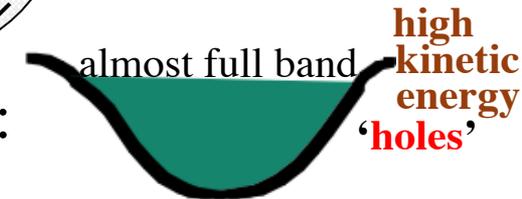
**How many different mechanisms of superconductivity do we need?**

# 1 mechanism for all materials: 'Hole superconductivity'

Superconductors **want to get rid of electrons** ==> expel electrons from interior to the surface

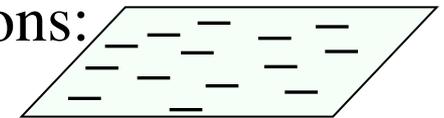


\* Which metals? those that have 'too many' electrons:



\* Which give highest  $T_c$ ? those that have most electrons:

*planes with negatively charged anions*



$(\text{CuO}_2)^-$  ;  $\text{B}^-$  in  $\text{MgB}_2$   
 $\text{As}^-$ ,  $\text{Se}^-$  in pnictides...

\* What does this explain? Meissner effect!

*Expulsion of magnetic field = expulsion of electrons*

\* What drives pairing, superconductivity and charge expulsion?

*Kinetic energy lowering*

Microscopics: 'Dynamic Hubbard models'

Macroscopics: new London-like equations

Electric field in interior

Charge current near surface ( $B \neq 0$ )

Spin current near surface ( $B = 0$ )

# 1989 Theory of hole superconductivity

References: <http://physics.ucsd.edu/~jorge/hole.html>

Collaborators: Frank Marsiglio; S. Tang, X. Q. Hong, H.Q. Lin

Key to the cuprates is  $O^=$  **Negatively charged anions**  
Key to Fe-As compounds is  $As^{3-}$   
Key to  $MgB_2$  is  $B^-$  **Hole conduction necessary**  
Key to  $H_2S$  is  $S^=$

- \* Hole polarizes the anion through which it conducts
- \* Superconductivity is driven by lowering of kinetic energy
- \* Alternative London electrodynamics

Models:

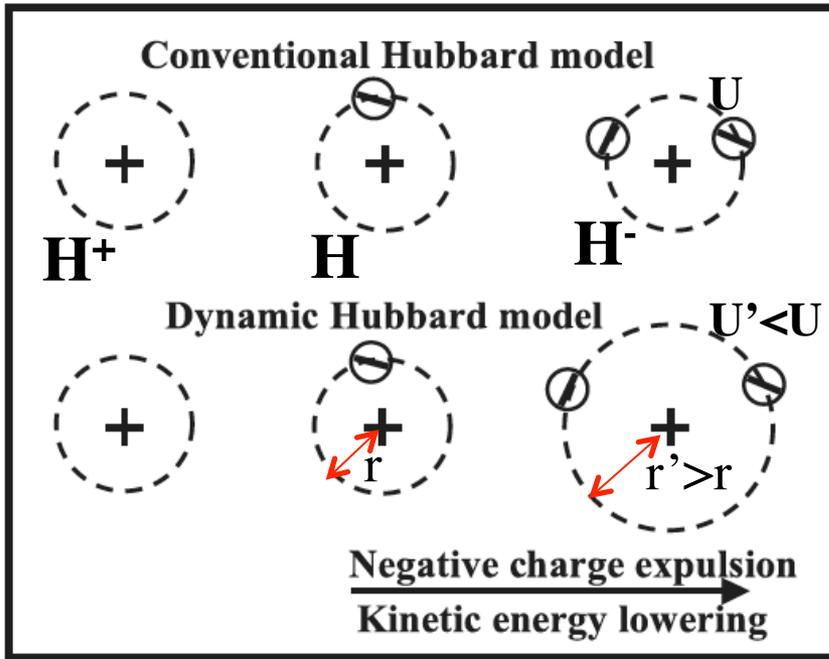
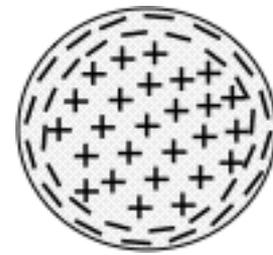
- \* Dynamic Hubbard models
- \* Electron-hole asymmetric electronic polarons
- \* Hubbard model with correlated hopping

Experimental support:

- \* Tunneling asymmetry (theory 1989, exp. 1995-2012)
- \* Optical sum rule 'violation' (theory 1992, exp. 1999-2012)
- \* Meissner effect (theory 2012-15, exp. 1933)

2015

# Microscopic physics: **Dynamic Hubbard model**



PRL 87, 206402 (2001)

PRB 87, 184506 (2013)

FIG. 1. In the conventional Hubbard model the atomic orbital is not modified by electronic occupancy. In the dynamic Hubbard model and in real atoms, addition of the second electron causes **orbital expansion** due to the electron-electron interaction. **Negative charge is expelled outward** and the **kinetic energy of the electrons is lowered** relative to that with a nonexpanded orbital.

$$K = \frac{\hbar^2}{2m_e r^2}$$

**Leads to, when band is almost full:**

- \* pairing and superconductivity
- \* negative charge expulsion from interior to surface
- \* tendency to charge inhomogeneity

**driven by  
kinetic energy  
lowering**

**site Hamiltonian:**

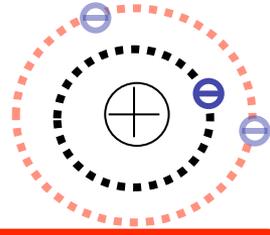
$$H_i = \frac{p_i^2}{2M} + \frac{1}{2} K q_i^2 + (U + \alpha q_i) n_{i\uparrow} n_{i\downarrow}$$

**lattice Hamiltonian:**

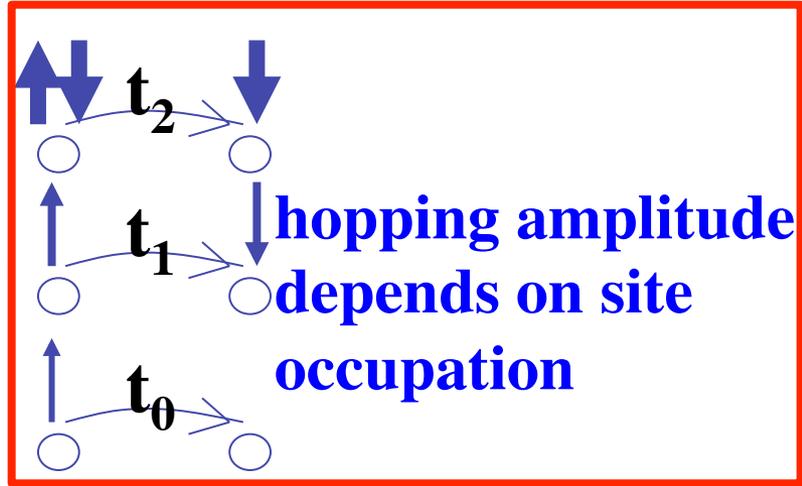
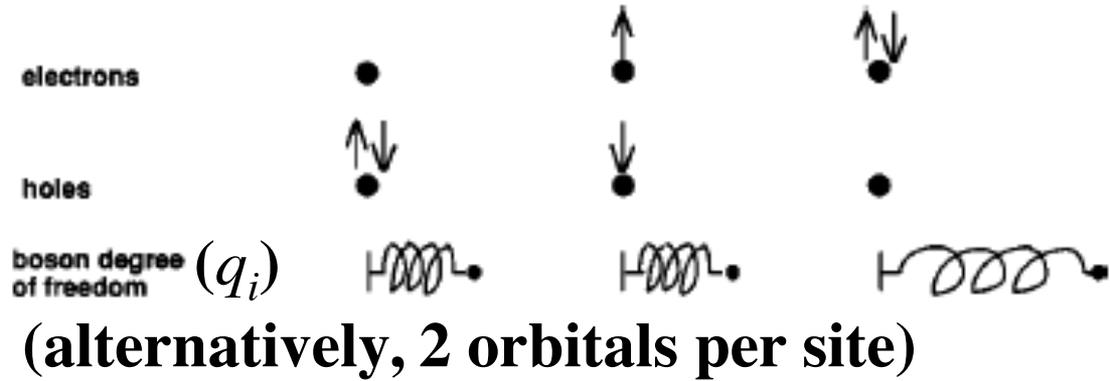
$$H = \sum_i H_i - \sum_{ij\sigma} t_{ij} [c_{i\sigma}^+ c_{j\sigma} + h.c.]$$

# Dynamic Hubbard models

$$H_i = \frac{p_i^2}{2M} + \frac{1}{2} K q_i^2 + (U + \alpha q_i) n_{i\uparrow} n_{i\downarrow}$$



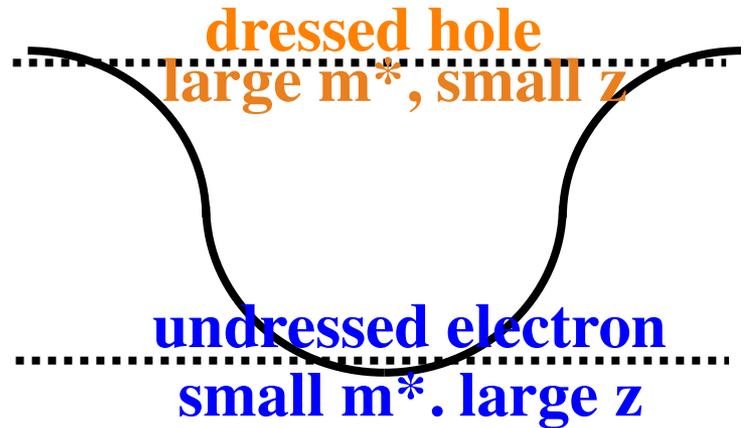
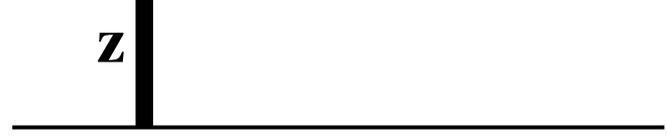
## Hubbard model + auxiliary boson degree of freedom



spectral function for hole creation



spectral function for electron creation

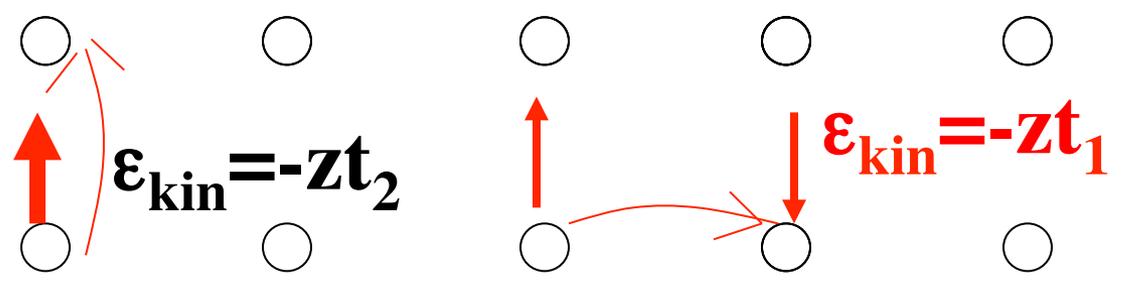
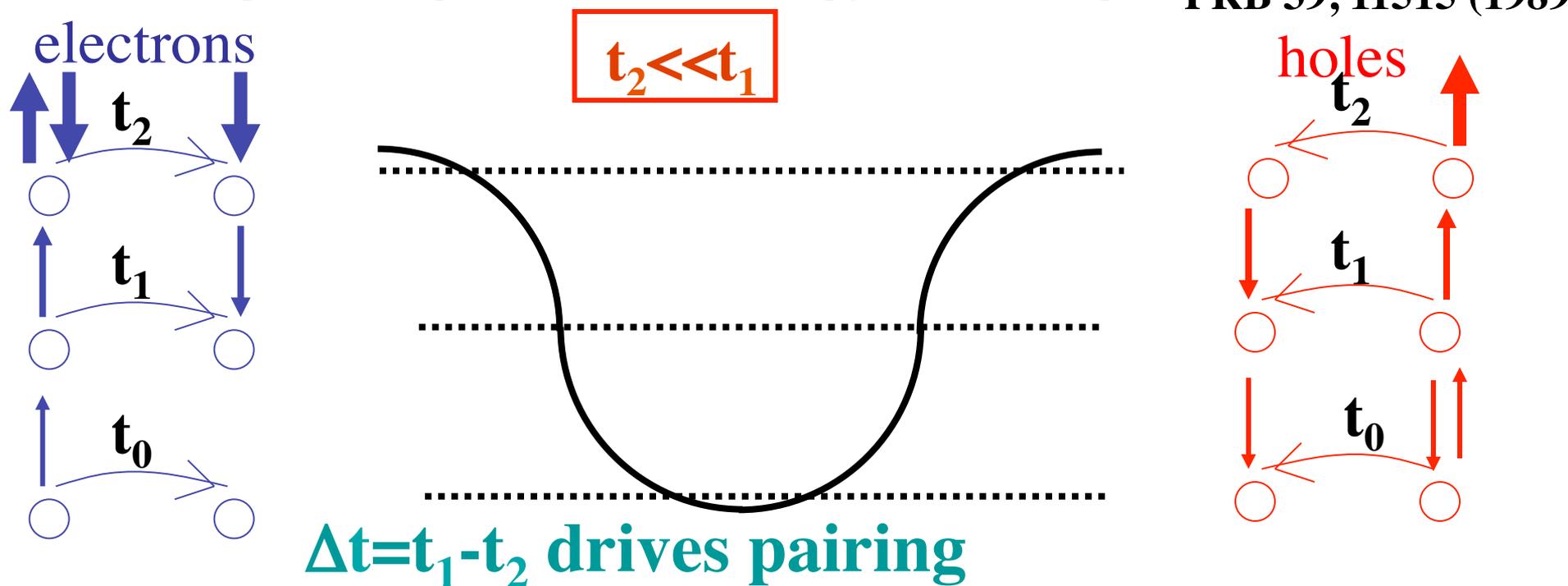


## Effective low energy Hamiltonian: Hubbard model with correlated hopping

$$H_{eff} \cong - \sum_{ij\sigma} [t_h + \Delta t(\tilde{n}_{i,-\sigma} + \tilde{n}_{j,-\sigma})] [\tilde{c}_{i\sigma}^+ \tilde{c}_{j\sigma} + h.c.] + U \sum_i \tilde{n}_{i\uparrow} \tilde{n}_{i\downarrow}$$

# Pairing through kinetic energy lowering

JEH, F. Marsiglio,  
PRB 39, 11515 (1989)



mobile 'bipolarons'  
pair mobility is  
larger than single  
carrier mobility

Effective low energy Hamiltonian:  
Hubbard model with correlated hopping

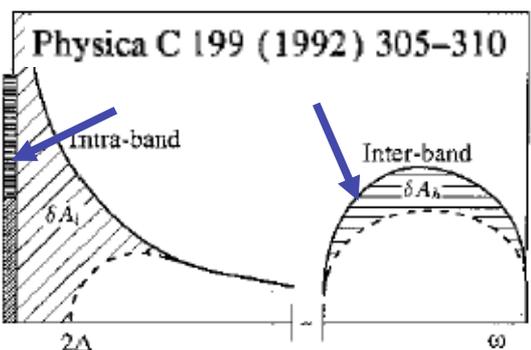
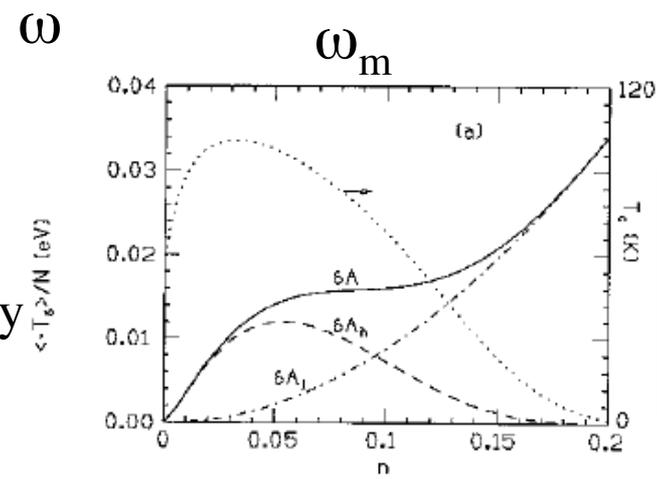
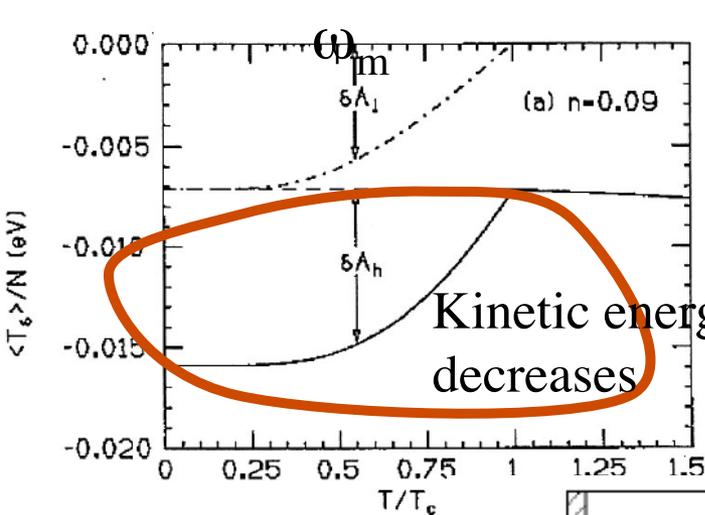
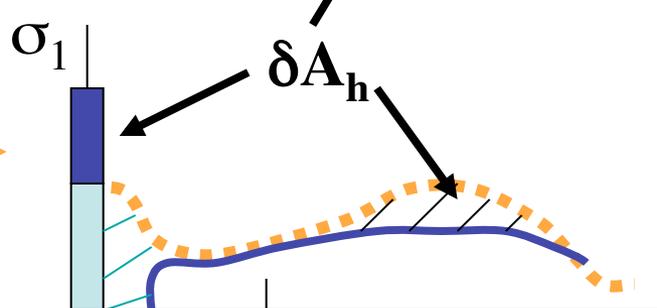
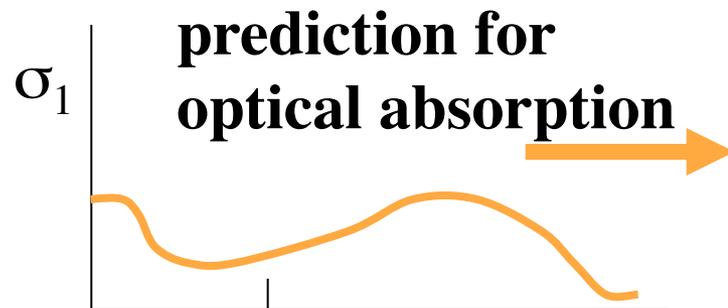
$$H_{\text{eff}} \cong - \sum_{ij\sigma} [t_h + \Delta t(\tilde{n}_{i,-\sigma} + \tilde{n}_{j,-\sigma})][\tilde{c}_{i\sigma}^+ \tilde{c}_{j\sigma} + h.c.] + U \sum_i \tilde{n}_{i\uparrow} \tilde{n}_{i\downarrow}$$

# Kinetic energy driven superconductivity (1992)

$$\langle -T_\delta \rangle = \sum_{i\sigma} \langle t_h + \Delta t (n_{i,-\sigma} + n_{i+\delta,-\sigma}) [c_{i\sigma}^+ c_{i+\delta,\sigma} + h.c.] \rangle \uparrow$$

$$\langle (t_h + \Delta t n_{i,-\sigma}) c_{i\sigma}^+ c_{j\sigma} \rangle = t(n) + \Delta t \langle c_{i\uparrow}^+ c_{i\downarrow}^+ \rangle \langle c_{i\downarrow} c_{j\uparrow} \rangle$$

$t_{\text{eff}}$  increases upon pairing



# Apparent violation of the conductivity sum rule in certain superconductors

Physica C 199 (1992) 305–310  
North-Holland

J.E. Hirsch

# Superconductors that change color when they become superconducting

Physica C 201 (1992) 347–361

It is pointed out that the Ferrell–Glover–Tinkham sum rule relating the “missing area” in the low frequency conductivity to the penetration depth can be violated in certain superconductors. Its breakdown indicates that the effective mass of the carriers changes in entering the superconducting state, and implies a change in the conductivity at frequencies much higher than the superconducting gap, possibly near infrared or visible. The model of hole superconductivity predicts the occurrence of this phenomenon.

$$\delta A_h = \frac{\pi e^2 a_s^2}{2\hbar^2} [\langle -T_s \rangle_s - \langle -T_s \rangle_n], \quad (11)$$

so that its existence indicates that the carriers change their kinetic energy on entering the superconducting state. Figure 1 shows schematically the expected behavior of the real part of the conductivity in the normal and superconducting states, as well as the two

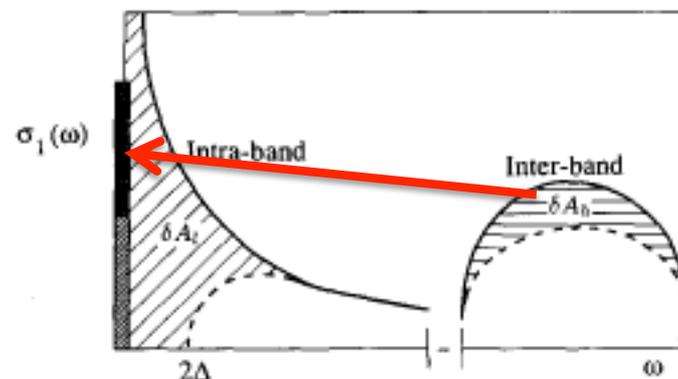


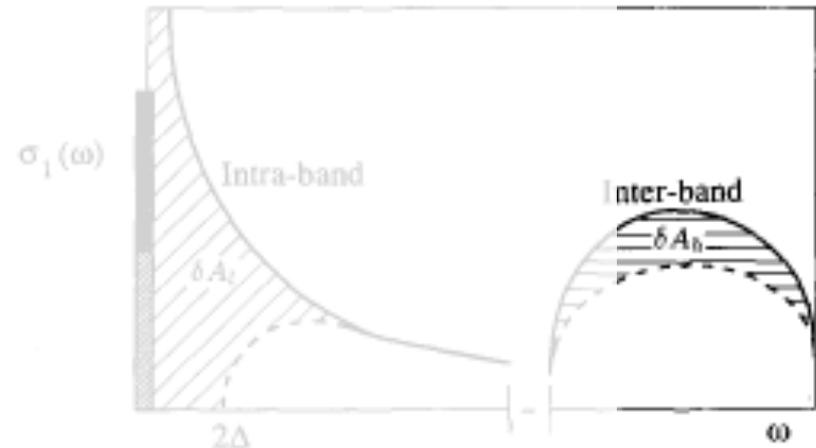
Fig. 1. Real part of the conductivity in the normal (solid lines) and superconducting (dashed lines) states (schematic). The missing areas from intra-band ( $\delta A_i$ , diagonally hatched) and inter-band ( $\delta A_b$ , horizontally hatched) transitions are shown as separate contributions to the  $\delta$ -function at zero frequency.

# Letters written to 6 optics experimentalists in 1992

SUMRULE1.LET;1 9-JUN-1992 15:03:39.78  
SUMRULE2.LET;1 9-JUN-1992 15:05:41.39  
SUMRULE4.LET;1 9-JUN-1992 15:08:13.60  
SUMRULE5.LET;1 9-JUN-1992 15:10:16.95  
SUMRULE6.LET;1 9-JUN-1992 15:14:21.22  
SUMRULE7.LET;1 9-JUN-1992 15:15:36.29

Dr. D. van der Marel  
Max Planck Institut  
Heisenbergstrasse 1  
W-700 Stuttgart 80  
Germany

June 10, 1992



Dear Dr. van der Marel:

I am enclosing a recent preprint with the following predictions for optical properties of a class of superconductors: i) the penetration depth (or imaginary conductivity) should be smaller (larger) than what is expected from the low frequency missing area in the frequency dependent conductivity, and ii) there should be a change in optical absorption at frequencies much higher than the superconducting energy gap on entering the superconducting state.

I hope these predictions can be experimentally tested; I believe them to apply particularly to the high Tc oxides (especially for low hole concentration, i.e. underdoped regime). I would very much appreciate if you would give me any comments on this.

Sincerely yours,

J.E. Hirsch

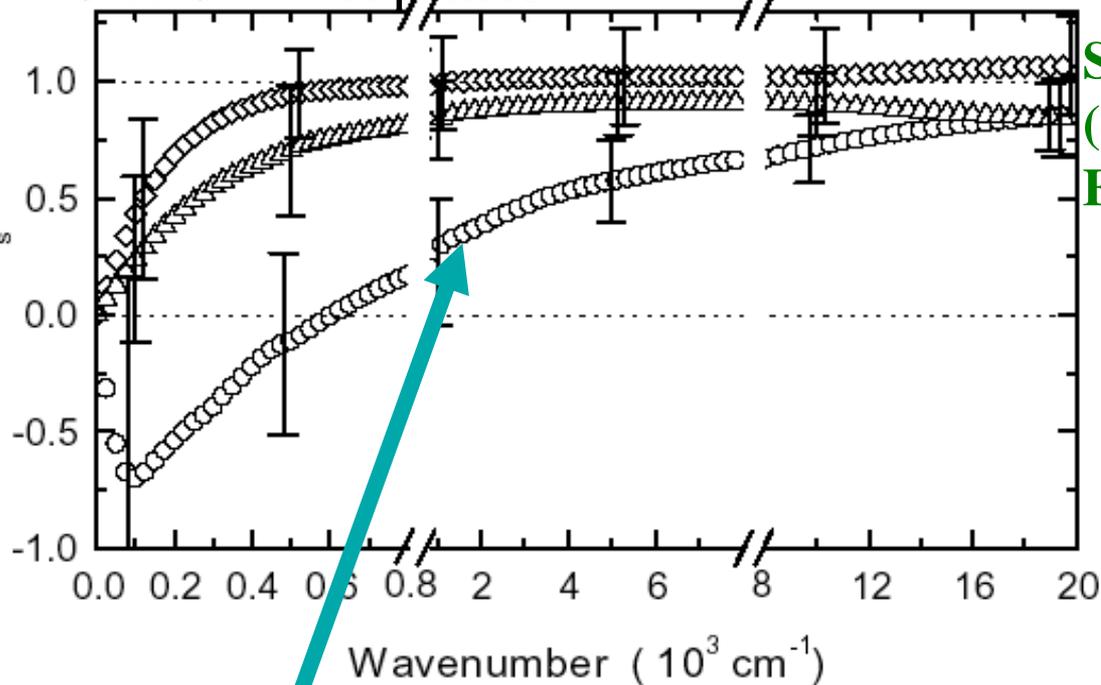
Optical sum rule

$$\Delta W = \int_0^{\omega_m} [\sigma_1^n(\omega) - \sigma_1^s(\omega)] d\omega$$

violation in cuprates:

kinetic energy

lowering  $\sim 1\text{meV}$



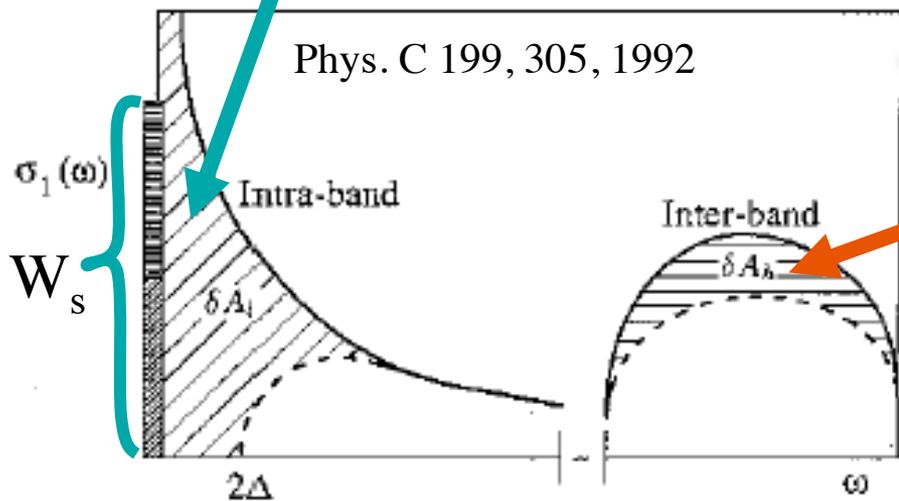
Santander et al

(cond-mat/0111539 (2001)

Europhys. Lett. 62, 568 (2003))

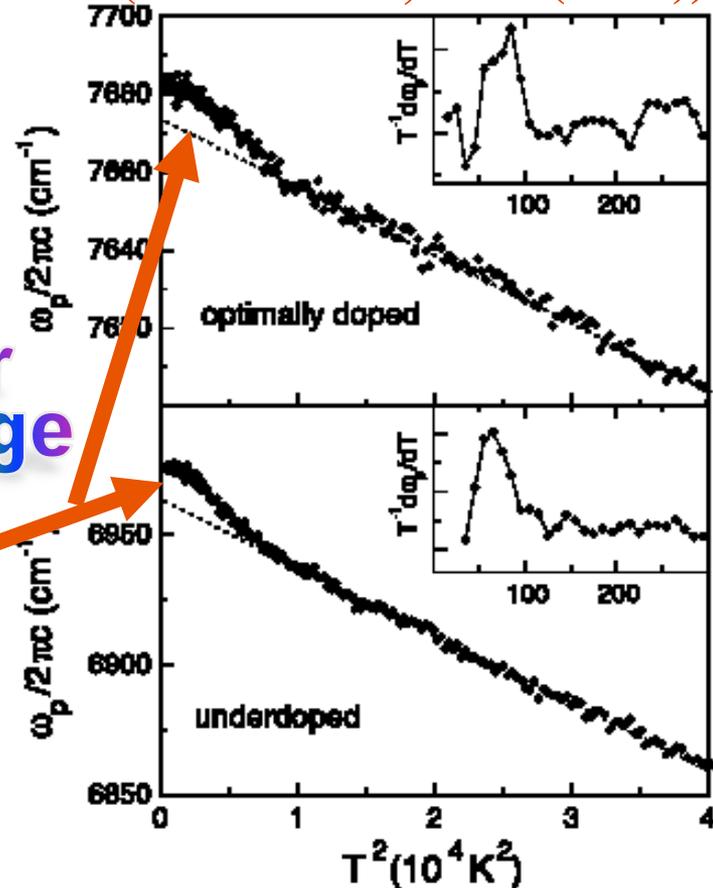
Van der Marel et al

(Science 295, 2239 (2002))



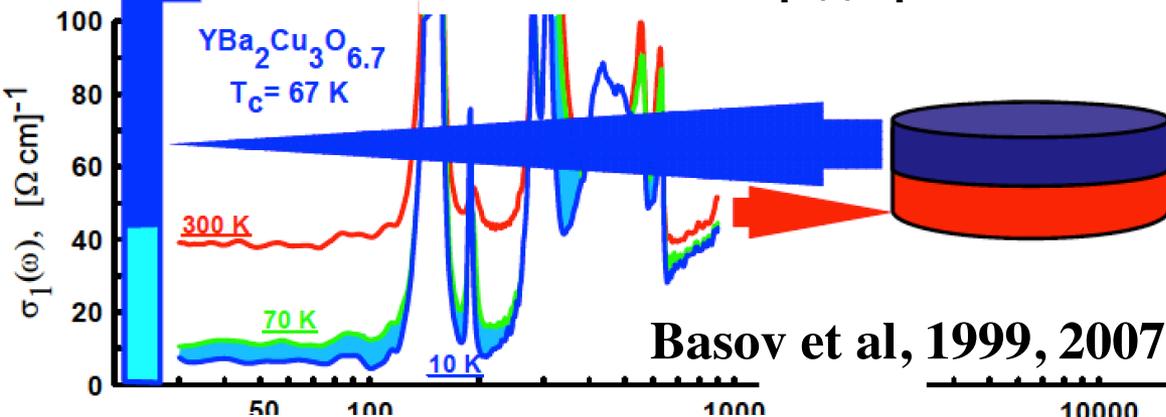
Phys. C 199, 305, 1992

color change



# c-axis pseudogap and c-axis condensate

[http://opticsumrule07.espci.fr/templates/d\\_basov.pdf](http://opticsumrule07.espci.fr/templates/d_basov.pdf)

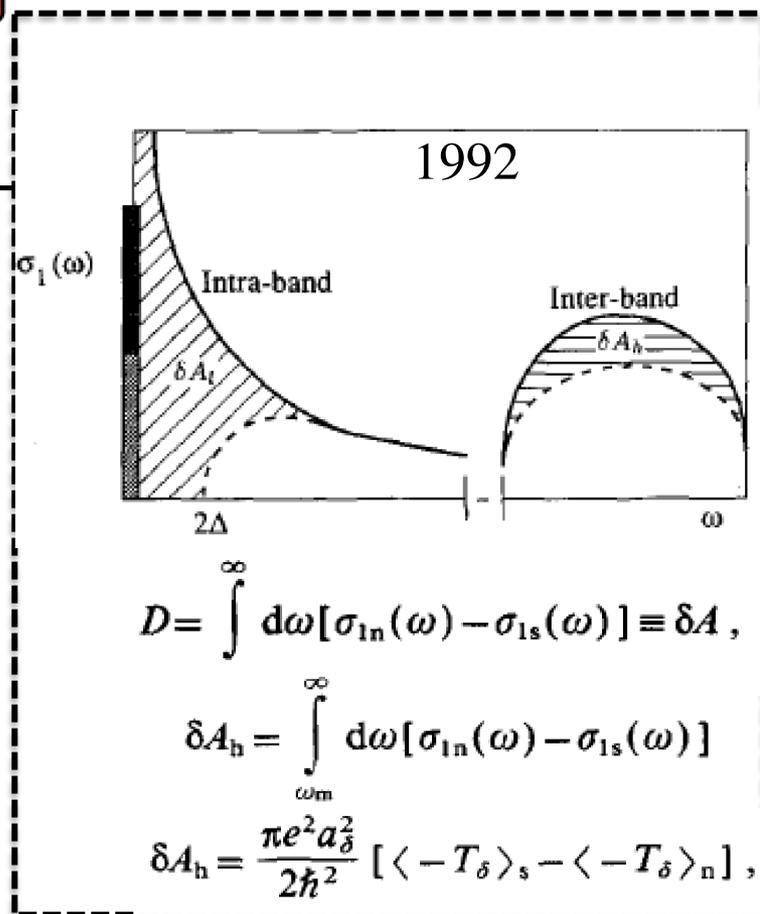


Basov et al, 1999, 2007

$$\rho_S = \int d\omega [\sigma_1^N(\omega) - \sigma_1^S(\omega)] + \Delta K$$

F-G-T sum rule

kinetic energy sum rule

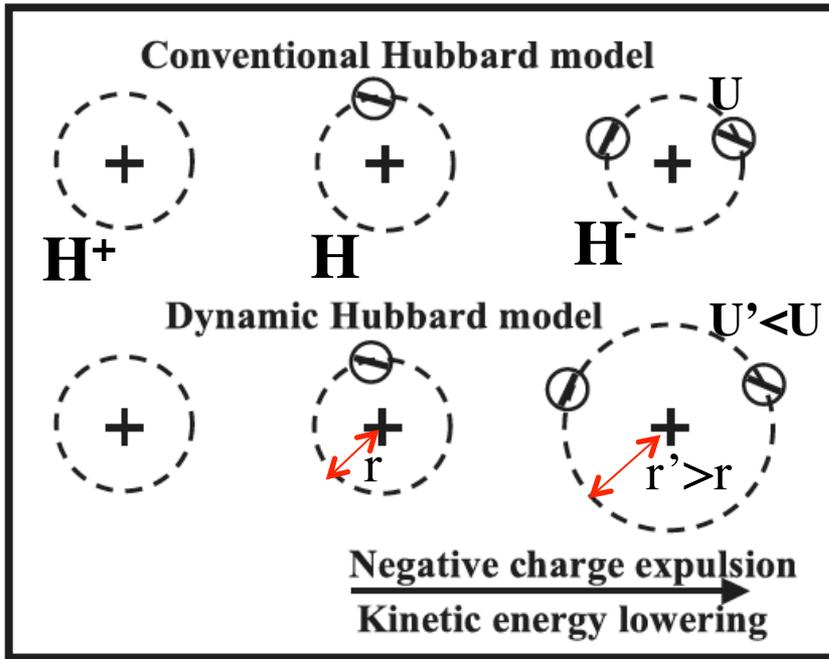
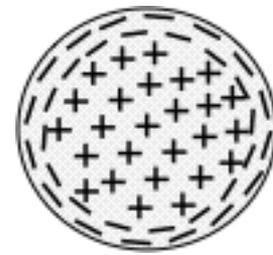


## Superconductivity-induced optical anomaly in an iron arsenide

A. Charnukha, P. Popovich, Y. Matiks, D. L. Sun, C. T. Lin,  
A. N. Yaresko, B. Keimer, A. V. Boris

(Submitted on 29 Sep 2010)

# Microscopic physics: **Dynamic Hubbard model**



PRL 87, 206402 (2001)

PRB 87, 184506 (2013)

FIG. 1. In the conventional Hubbard model the atomic orbital is not modified by electronic occupancy. In the dynamic Hubbard model and in real atoms, addition of the second electron causes **orbital expansion** due to the electron-electron interaction. **Negative charge is expelled outward** and the **kinetic energy of the electrons is lowered** relative to that with a nonexpanded orbital.

$$K = \frac{\hbar^2}{2m_e r^2}$$

**Leads to, when band is almost full:**

- \* pairing and superconductivity
- \* **negative charge expulsion from interior to surface**
- \* tendency to charge inhomogeneity

**driven by  
kinetic energy  
lowering**

**Effective low energy Hamiltonian:**

**Hubbard model with correlated hopping**

$$H_{eff} \cong - \sum [t_h + \Delta t(\tilde{n}_{i,-\sigma} + \tilde{n}_{j,-\sigma})][\tilde{c}_{i\sigma}^+ \tilde{c}_{j\sigma} + h.c.] + U \sum \tilde{n}_{i\uparrow} \tilde{n}_{i\downarrow}$$

# Negative charge expulsion in dynamic Hubbard model

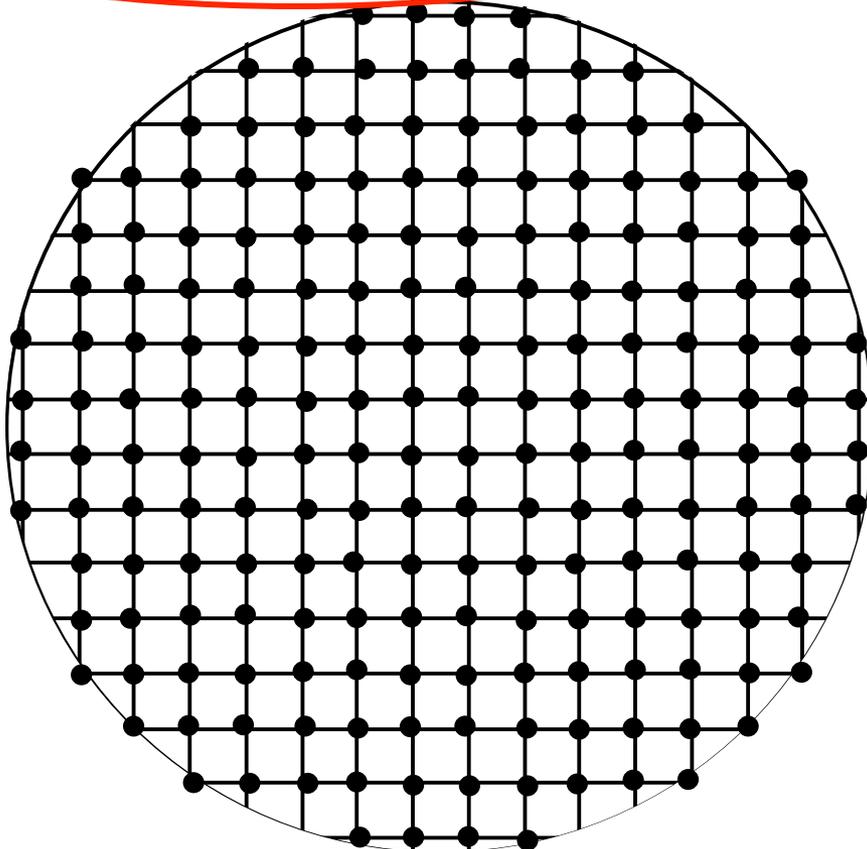
PRB 87, 184506 (2013)



$$H = - \sum_{ij\sigma} t_{ij}^{\sigma} [c_{i\sigma}^{\dagger} c_{j\sigma} + \text{H.c.}] + U \sum_i n_{i\uparrow} n_{i\downarrow},$$

$$t_{ij}^{\sigma} = t_h + \Delta t (n_{i,-\sigma} + n_{j,-\sigma}) + \Delta t_2 n_{i,-\sigma} n_{j,-\sigma}.$$

kinetic energy **decreases**  
with increasing hole occu-  
pation



→ system wants to have more  
holes in the interior

→ expels electrons from interior  
to the surface

(fewer nearest neighbors at  
the surface)

# Negative charge expulsion in dynamic Hubbard model

$$H = - \sum_{ij\sigma} t_{ij}^\sigma [c_{i\sigma}^\dagger c_{j\sigma} + \text{H.c.}] + U \sum_i n_{i\uparrow} n_{i\downarrow},$$

$$t_{ij}^\sigma = t_h + \Delta t (n_{i,-\sigma} + n_{j,-\sigma}) + \Delta t_2 n_{i,-\sigma} n_{j,-\sigma}.$$

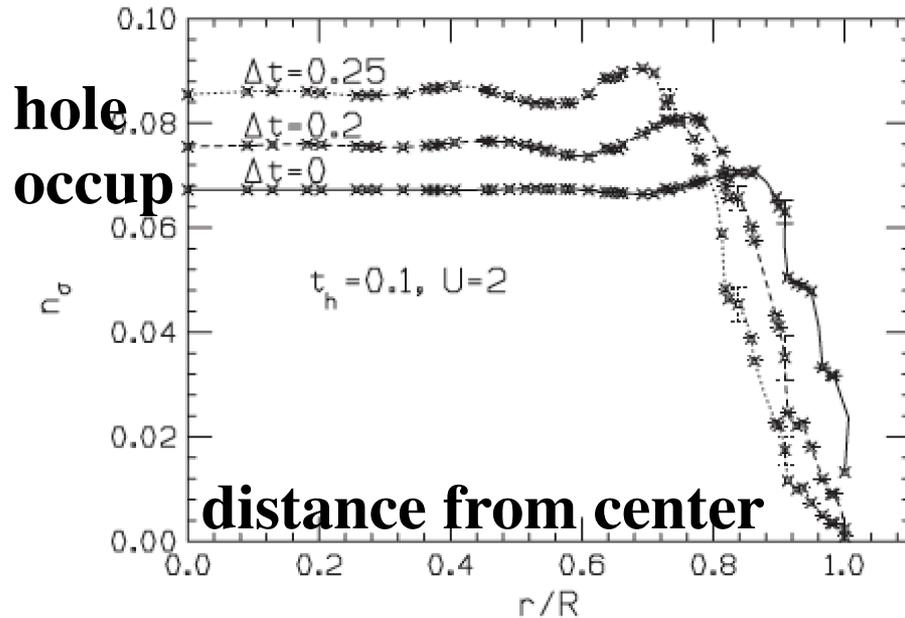


FIG. 3. Hole site occupation per spin for a cylinder of radius  $R = 11$  as a function of  $r/R$ , with  $r$  the distance to the center, for a cubic lattice of side length 1. There are 377 sites in a cross-sectional area ( $\pi R^2 = 380.1$ ). The average occupation (both spins) is  $n = 0.126$  hole per site and the temperature is  $k_B T = 0.02$ .

$$t(n_h) = t_h + n_h \Delta t$$

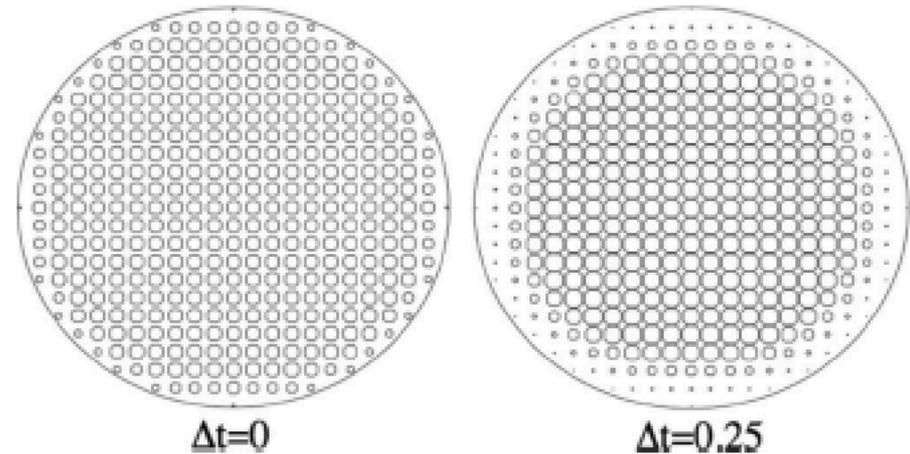


FIG. 4. Diameters of the circles are proportional to the hole occupation at the site. Note that for finite  $\Delta t$  the hole occupation increases in the interior and is depleted near the surface. Parameters correspond to the cases shown in Fig. 3.

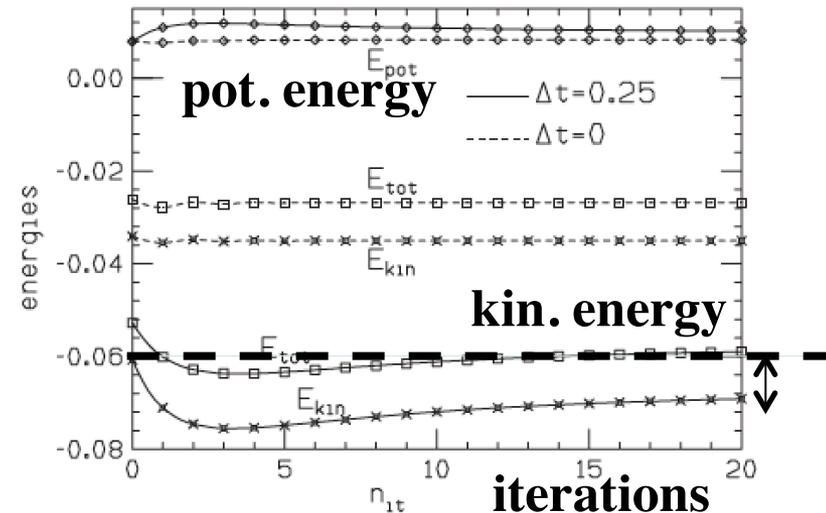


FIG. 5. Kinetic, potential, and total energy per site for  $\Delta t = 0.25$  as a function of the number of iterations starting with a uniform hole distribution.

## For larger $\Delta t$ : phase separation

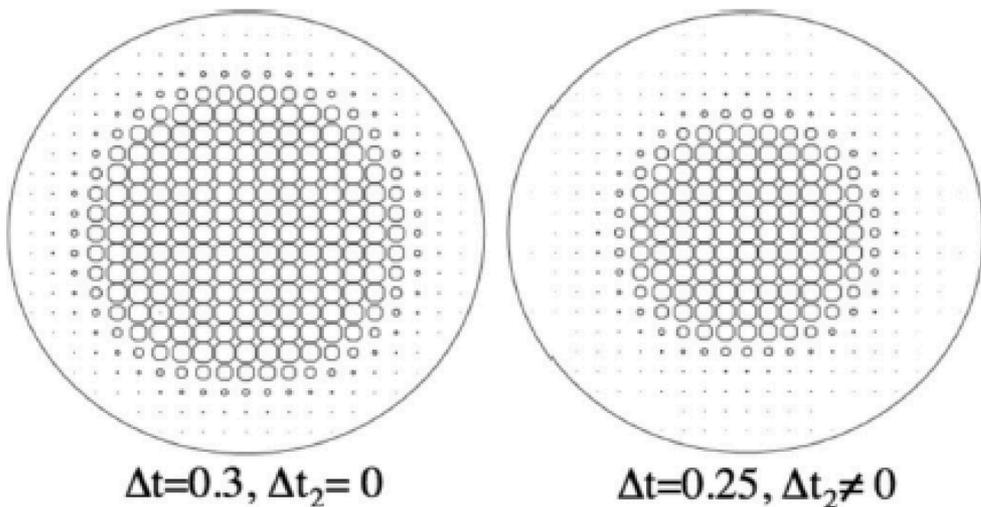


FIG. 8. As the correlated hopping terms increase, the system develops a tendency toward phase separation, where essentially all the holes condense to the interior. Parameters are the same as in Fig. 3 except as indicated. The maximum hole occupation per spin is 0.128 (left) and 0.214 (right); the average hole occupation per spin is 0.063.

Consider a flat density of states for simplicity. The effective bandwidth increases linearly with  $n$ ,

$$D(n) = D_h + Kn, \quad (16)$$

with  $D = 2zt$ ,  $D_h = 2zt_h$ ,  $K = 2z\Delta t$ . The density of states per site per spin is given by  $g(\epsilon) = 1/D$ , and the ground-state kinetic energy by

$$E_{\text{kin}} = \int_{-D/2}^{\mu} \epsilon g(\epsilon) d\epsilon = \frac{D}{4}(n^2 - 2n), \quad (17)$$

with  $\mu = (n - 1)D(n)/2$  the chemical potential for  $n$  holes per site. Adding the on-site repulsion in a mean-field approximation yields

$$E_o(n) = \frac{D_h + nK}{4}(n^2 - 2n) + \frac{U}{4}n^2, \quad (18)$$

and the system will be unstable towards phase separation into hole-rich and hole-poor regions when the condition

$$\frac{\partial^2 E_0}{\partial n^2} = \frac{U + D_h}{2} + K \left( \frac{3}{2}n - 1 \right) < 0 \quad (19)$$

is satisfied, hence

$$K > \frac{U + D_h}{2(1 - \frac{3}{2}n)} \quad (20)$$

or, equivalently,

$$\Delta t > \frac{t_h + U/(2z)}{2(1 - \frac{3}{2}n)}. \quad (21)$$

# microscopic inhomogeneity

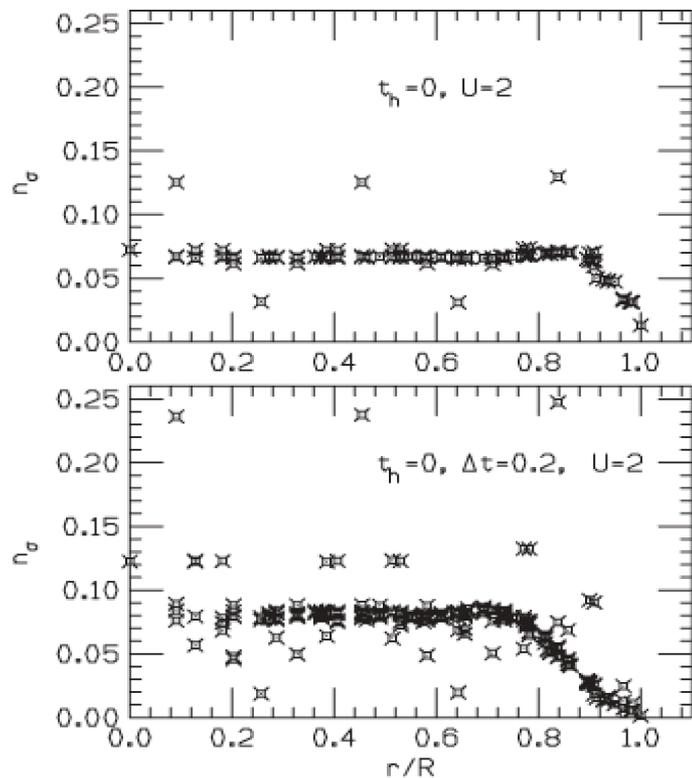


FIG. 10. Hole site occupation per spin in a system of radius  $R = 11$  with five impurities, at positions  $(-1, 0)$ ,  $(2, 2)$ ,  $(3, -4)$ ,  $(-5, -5)$ , and  $(-6, 7)$ , and potential strength  $-0.2, +0.2, -0.2, +0.2$ , and  $-0.2$ , respectively. Note the much larger variation in densities generated in the dynamic Hubbard model (lower panel;  $\Delta t_2 \neq 0$ ) than in the conventional Hubbard model (upper panel). Average hole occupation per site is  $n = 0.126$ .

# negatively charged grain boundaries

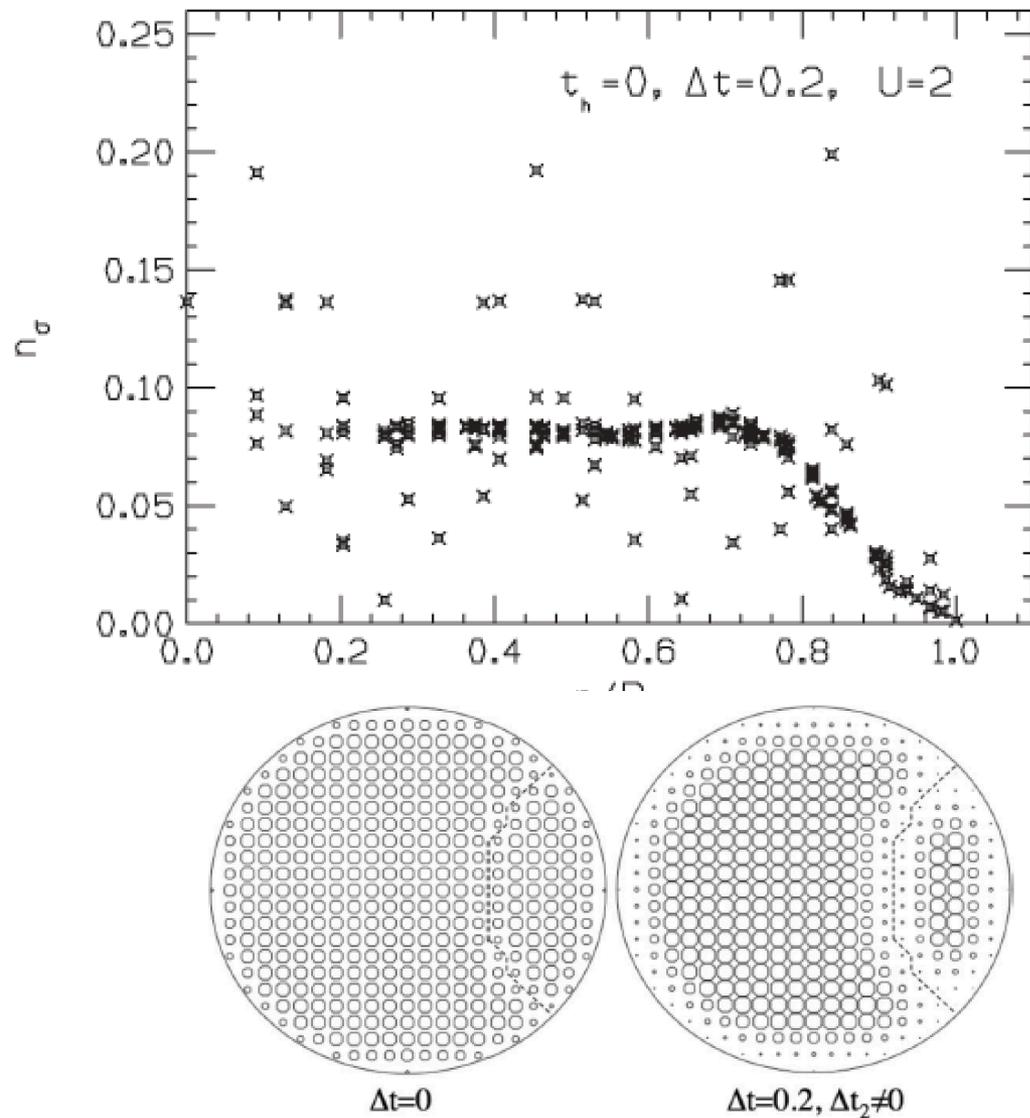
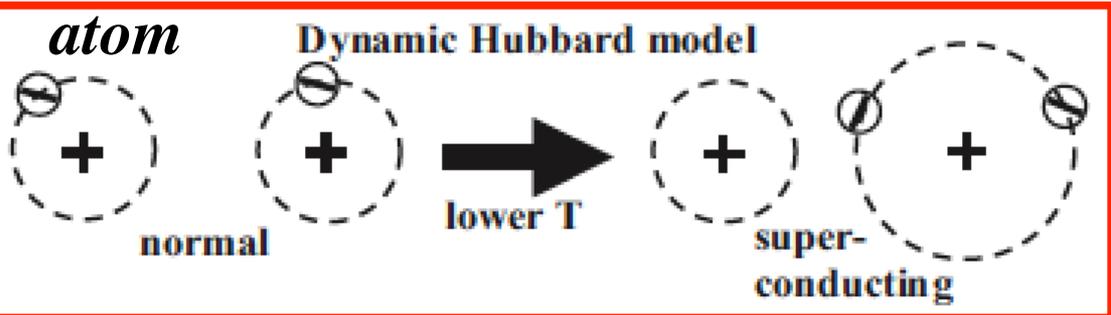


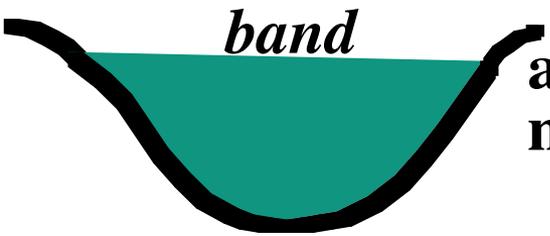
FIG. 14. Effect of a grain boundary, indicated by the dashed line for the conventional and dynamic Hubbard models with  $t_h = 0.1$ ,  $U = 2$ . We assume that the hopping amplitude is reduced by a factor of 0.3 for sites on opposite sides of the grain boundary. The hole occupation is depleted in the vicinity of the grain boundary in both cases, however, the effect is much larger and extends over a wider range for the dynamic Hubbard model than for the conventional one.

# Transition from normal to superconducting state:

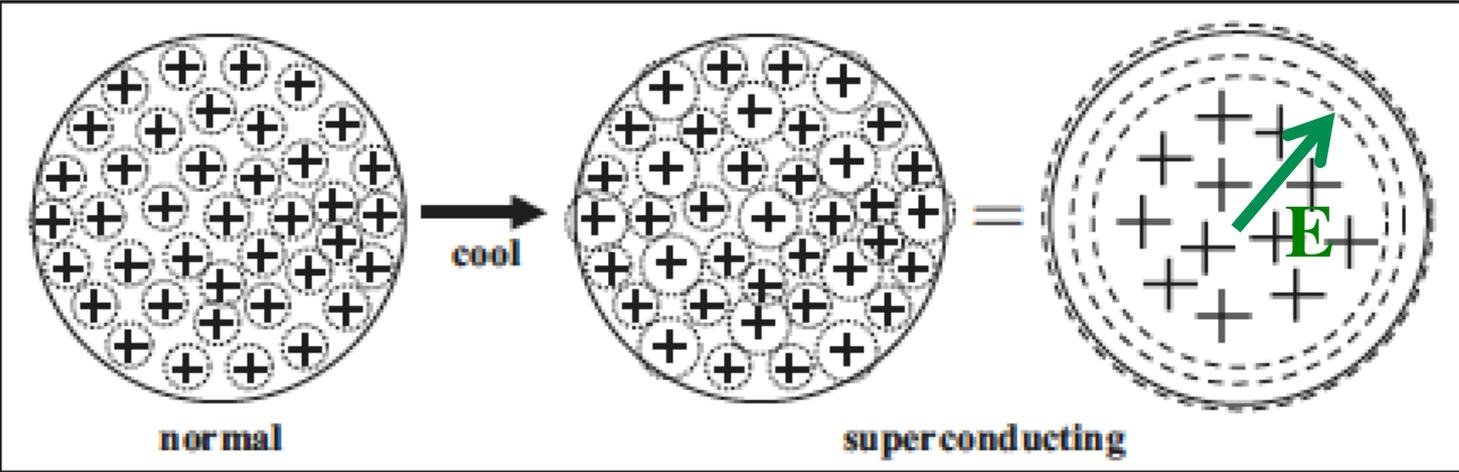


- \* pairing
- \* orbits become larger
- \* kinetic energy is lowered
- \* negative charge is expelled

$$K = \frac{\hbar^2}{2m_e r^2}$$



almost full band → many electrons, high kin.energy  
 negative ions → a lot of negative charge  
 → system expels electrons



macroscopic charge inhomogeneity

An outward-pointing electric field exists in the interior of superconductors at zero temperature

# Electrodynamic equations for 'giant atom'

The Electromagnetic Equations of the Supraconductor (1935)

SUPRALEITUNG UND DIAMAGNETISMUS (1935)

von F. und H. LONDON

diese noch unbekannt Koppelung eine Verfestigung der Elektronenwellenfunktion in erster Näherung auch gegen elektrische Störungen zustande bringt <sup>1)</sup>. Dann würde man zu einer Gleichung

$\rho = \text{charge density}$   $\rho = - \frac{e^2 n}{mc^2} \varphi$   $\varphi = \text{electric potential}$  (12b)

1) z.B. dürfte dies wohl zutreffen für den Fall, dass durch eine starke Elektron-Ion-Koppelung die Elektron-Ionengitter-Koppelung zu einem Effekt 2. Ordnung dert wird.

**'rigidity' against electric perturbations**

# Electrodynamic equations for 'giant atom'

The Electromagnetic Equations of the Supraconductor (1935)

SUPRALEITUNG UND DIAMAGNETISMUS (1935)

von F. und H. LONDON

diese noch unbekannte Koppelung eine Verfestigung der Elektronenwellenfunktion in erster Näherung auch gegen elektrische Störungen zustande bringt <sup>1)</sup>. Dann würde man zu einer Gleichung

$\rho = \text{charge density}$   $\rho = - \frac{e^2 n}{mc^2} \varphi$   $\varphi = \text{electric potential}$  (12b)

An Experimental Examination of the Electrostatic Behaviour of Supraconductors (1936)

By H. LONDON, Clarendon Laboratory, Oxford

It follows from these measurements that no electrostatic fields exist in a pure supraconductor, not even in a thin surface layer, at least to the approximation to which this is true for normal conductors. Accordingly

## Derivation of conventional London equation:

$$J = nev \quad (\mathbf{n}=\text{density, } \mathbf{v}=\text{speed, } \mathbf{J}=\text{current})$$

$$m \frac{dv}{dt} = eE \implies \frac{\partial J}{\partial t} = \frac{ne^2}{m} E \quad \text{free acceleration of electrons}$$

$$\frac{\partial J}{\partial t} = \frac{ne^2}{m} E \implies \frac{\partial}{\partial t} \nabla \times J = \frac{ne^2}{m} \nabla \times E = -\frac{ne^2}{mc} \frac{\partial B}{\partial t}$$

Integrate, ignore integration constant, gives London eq.

$$\nabla \times J = -\frac{ne^2}{mc} B, \text{ with } \nabla \times B = \frac{4\pi}{c} J \implies \nabla^2 B = \frac{1}{\lambda_L^2} B = \frac{4\pi ne^2}{mc^2} B$$

$$J = -\frac{ne^2}{mc} A$$

$$\vec{\nabla} \cdot \vec{A} = 0$$

# Derivation of conventional London equation:

$J = nev$  (n=density, v=speed, J=current)

$m \frac{dv}{dt} = eE \implies \frac{\partial J}{\partial t} = \frac{ne^2}{m} E$  free acceleration of electrons

$\frac{\partial J}{\partial t} = \frac{ne^2}{m} E \implies \frac{\partial}{\partial t} \nabla \times J = \frac{ne^2}{m} \nabla \times E = -\frac{ne^2}{mc} \frac{\partial B}{\partial t}$

Integrate, ignore integration constant, gives London eq.

$\nabla \times J = -\frac{ne^2}{mc} B$ , with  $\nabla \times B = \frac{4\pi}{c} J \implies \nabla^2 B = \frac{1}{\lambda_L^2} B = \frac{4\pi ne^2}{mc^2} B$

$J = -\frac{ne^2}{mc} A$

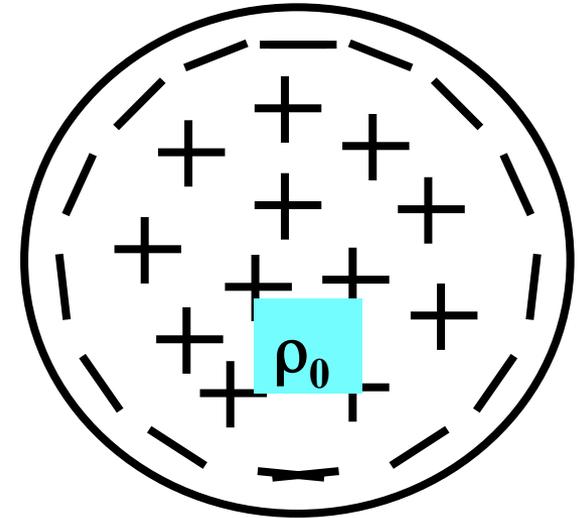
$\vec{\nabla} \cdot \vec{A} = 0$

Note:  $\frac{\partial J}{\partial t} = -\frac{ne^2}{mc} \frac{\partial A}{\partial t}$ ,  $E = -\nabla\phi - \frac{1}{c} \frac{\partial A}{\partial t} \implies \frac{\partial J}{\partial t} = \frac{ne^2}{m} (E + \nabla\phi)$

# New London-like equations for superconductors (JEH, PRB69, 214515(2004))

$$1) \quad J = -\frac{ne^2}{mc} A = -\frac{c}{4\pi\lambda_L^2} A \quad ; \quad \frac{1}{\lambda_L^2} \equiv \frac{4\pi ne^2}{mc^2}$$

$$2) \quad \nabla \cdot A + \frac{1}{c} \frac{\partial \phi}{\partial t} = 0 \quad ; \quad (\text{Lorenz gauge})$$



$$\nabla \cdot J = -\frac{c}{4\pi\lambda_L^2} \nabla \cdot A \quad , \quad \text{continuity equation: } \nabla \cdot J + \frac{\partial \rho}{\partial t} = 0 \quad \implies$$

$$\frac{\partial \rho}{\partial t} = -\frac{1}{4\pi\lambda_L^2} \frac{\partial \phi}{\partial t}$$

integrate in time, 1 integration constant  $\rho_0$  , ...

$$\implies \quad \rho(r,t) - \rho_0 = -\frac{1}{4\pi\lambda_L^2} [\phi(r,t) - \phi_0(r)] \quad \Bigg| \quad \phi_0(r) = \int d^3 r' \frac{\rho_0}{|r - r'|}$$

## Electrostatics:

(JEH, PRB69, 214515(2004))

$$\nabla^2(\phi(r) - \phi_0(r)) = \frac{1}{\lambda_L^2}(\phi(r) - \phi_0(r))$$

$$\nabla^2(\rho(r) - \rho_0) = \frac{1}{\lambda_L^2}(\rho(r) - \rho_0)$$

$$\nabla^2\phi(r) = -4\pi\rho(r) \quad \nabla^2\phi_0(r) = -4\pi\rho_0$$

$$\nabla^2(\vec{E} - \vec{E}_0) = \frac{1}{\lambda_L^2}(\vec{E} - \vec{E}_0)$$

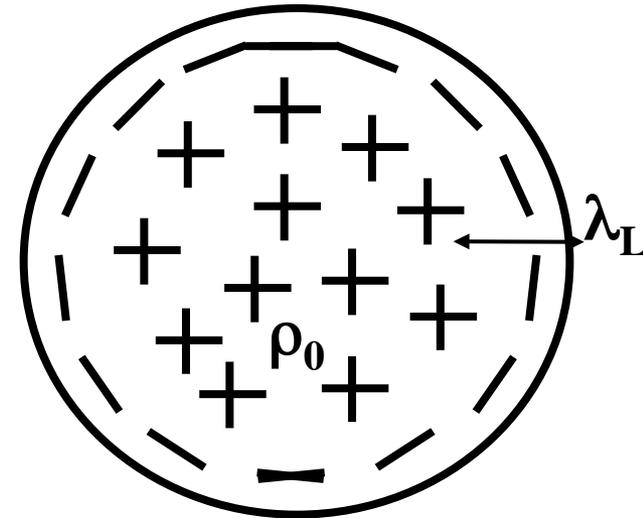
$$\nabla^2\phi(r) = 0 \quad \text{outside supercond.}$$

+assume  $\phi(r)$  and its normal derivative are continuous at surface

Solution for sphere of radius R:

$$\rho(r) = \rho_0 \left( 1 - \frac{R^3}{3\lambda_L^2} \frac{\sinh(r/\lambda_L)}{R/\lambda_L \cosh(R/\lambda_L) - \sinh(R/\lambda_L)} \right)$$

$$\vec{E}(r) = \frac{4}{3}\pi\rho_0 \left[ 1 - \frac{R^3}{r^3} \frac{r/\lambda_L \cosh(r/\lambda_L) - \sinh(r/\lambda_L)}{R/\lambda_L \cosh(R/\lambda_L) - \sinh(R/\lambda_L)} \right] \vec{r}$$



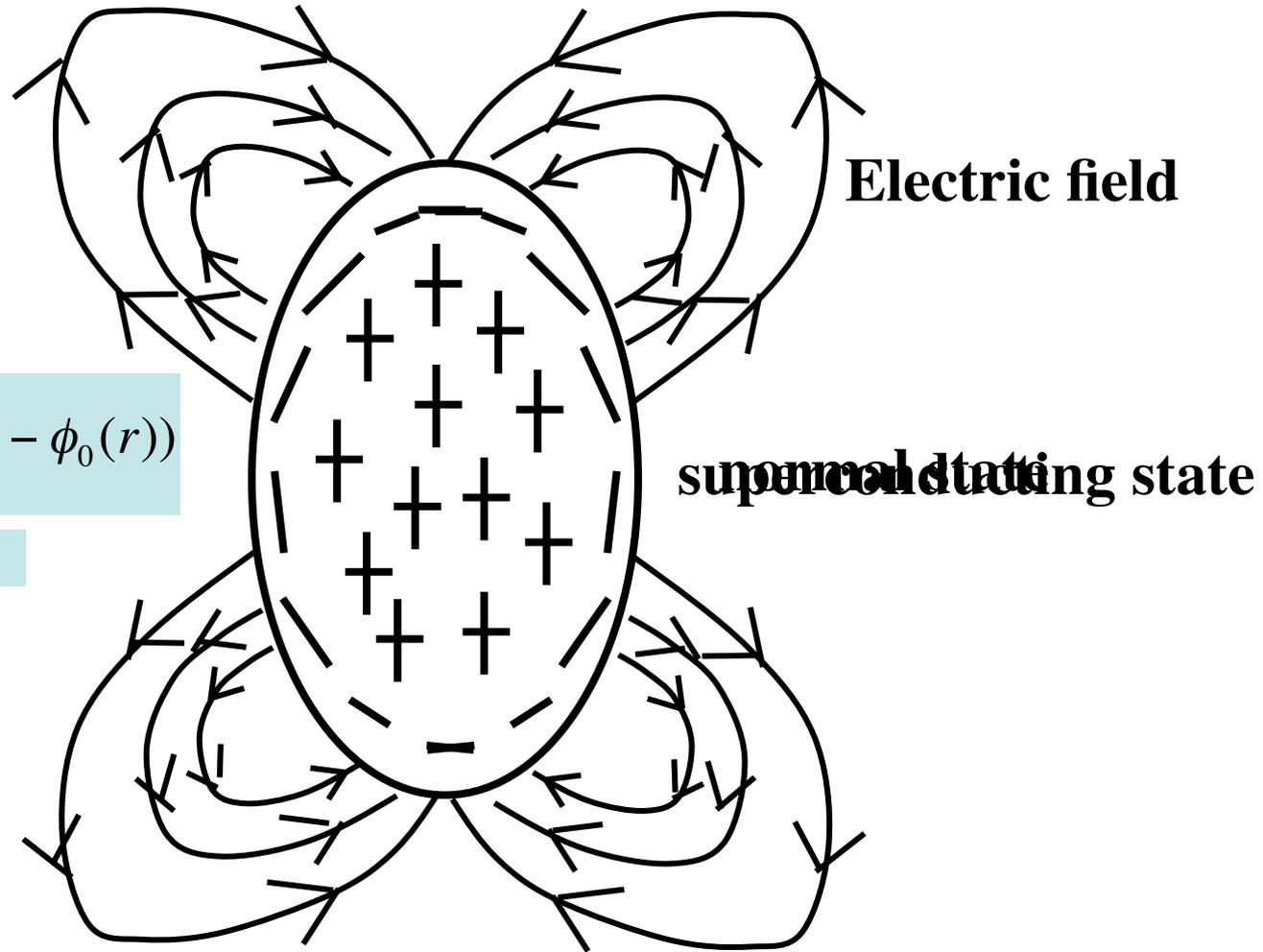
**No electric field outside sphere**

## Elliptical shape

$$\nabla^2(\phi(r) - \phi_0(r)) = \frac{1}{\lambda_L^2}(\phi(r) - \phi_0(r))$$

$$\nabla^2\phi_0(r) = -4\pi\rho_0 \quad \text{inside}$$

$$\nabla^2\phi(r) = 0 \quad \text{outside}$$



**test experimentally by measuring electric fields in the neighborhood of superconducting small particles**

# Electrostatics:

$$\nabla^2(\phi(r) - \phi_0(r)) = \frac{1}{\lambda_L^2}(\phi(r) - \phi_0(r))$$

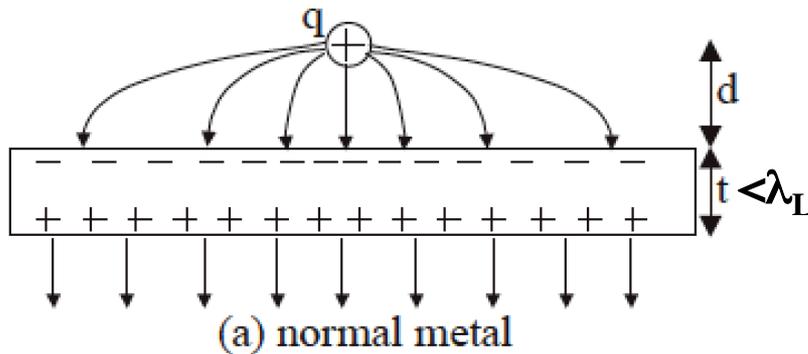
$$\nabla^2(\rho(r) - \rho_0) = \frac{1}{\lambda_L^2}(\rho(r) - \rho_0)$$

$$\nabla^2\phi(r) = -4\pi\rho(r) \quad \nabla^2\phi_0(r) = -4\pi\rho_0$$

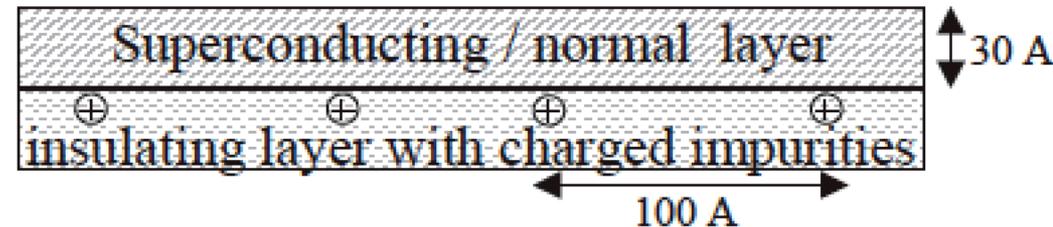
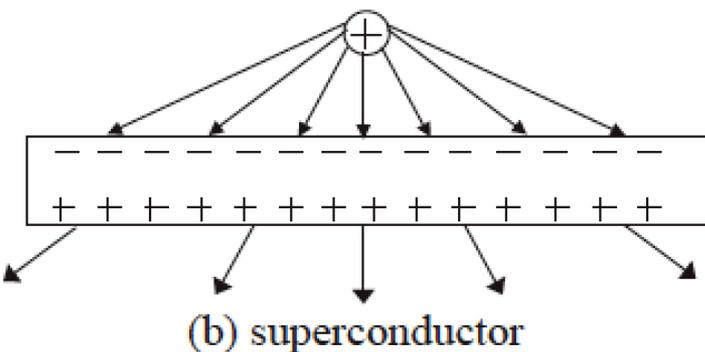
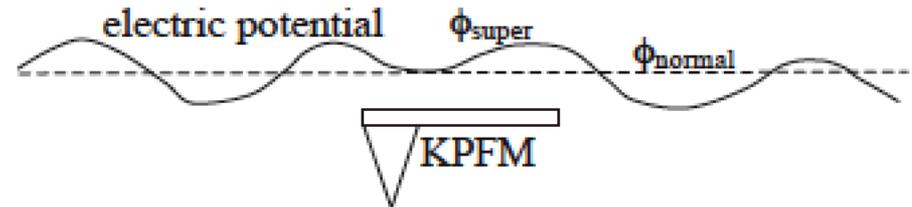
$$\nabla^2(\vec{E} - \vec{E}_0) = \frac{1}{\lambda_L^2}(\vec{E} - \vec{E}_0)$$

$$\nabla^2\phi(r) = 0$$

electric screening length is  $\lambda_L$



**Experiment to test it**



**Physica C 508, 21 (2015)**

# Meissner effect:

## London equations *do not explain the Meissner effect!*

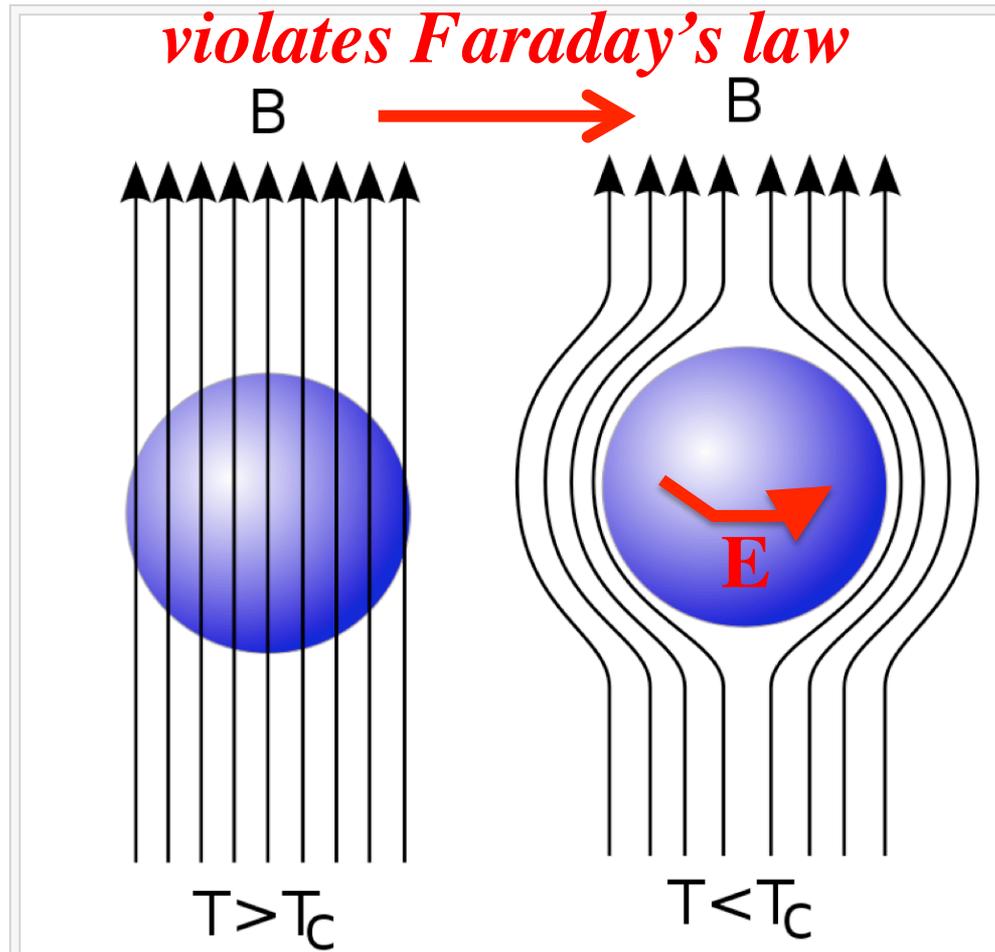
From Wikipedia, the free encyclopedia

The **London equations**, developed by brothers [Fritz](#) and [Heinz London](#) in 1935,<sup>[1]</sup> relate current to [electromagnetic fields](#) in and around a [superconductor](#). Arguably the simplest meaningful description of superconducting phenomena, they form the genesis of almost any modern introductory text on the subject.<sup>[2][3][4]</sup> A major triumph of the equations is their ability to explain the [Meissner effect](#),<sup>[5]</sup> wherein a material exponentially expels all internal magnetic fields as it crosses the superconducting threshold.

**Contents** [\[hide\]](#)

[1 Formulations](#)

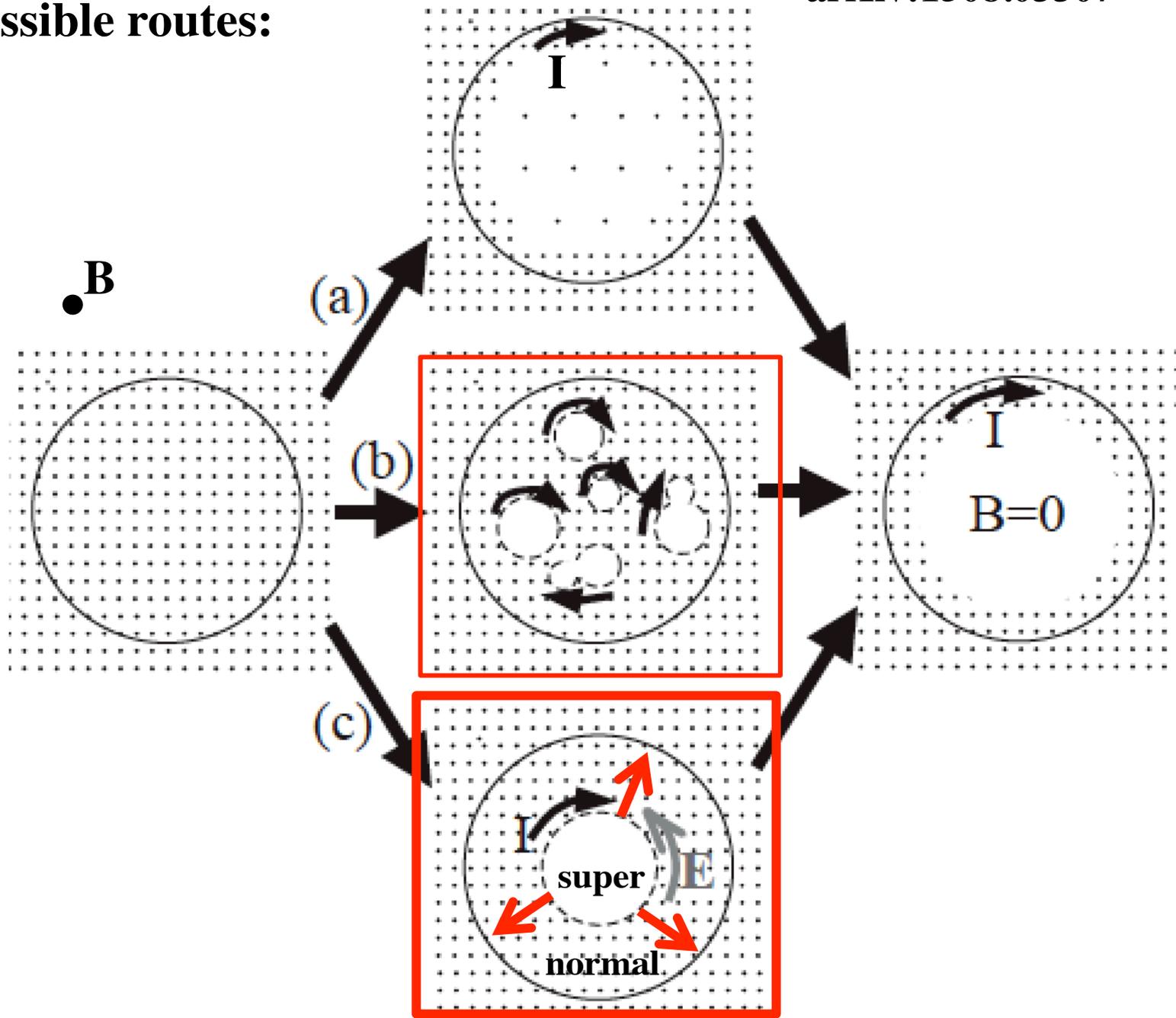
[2 London penetration depth](#)



**BCS *does not explain the process* by which B is expelled**

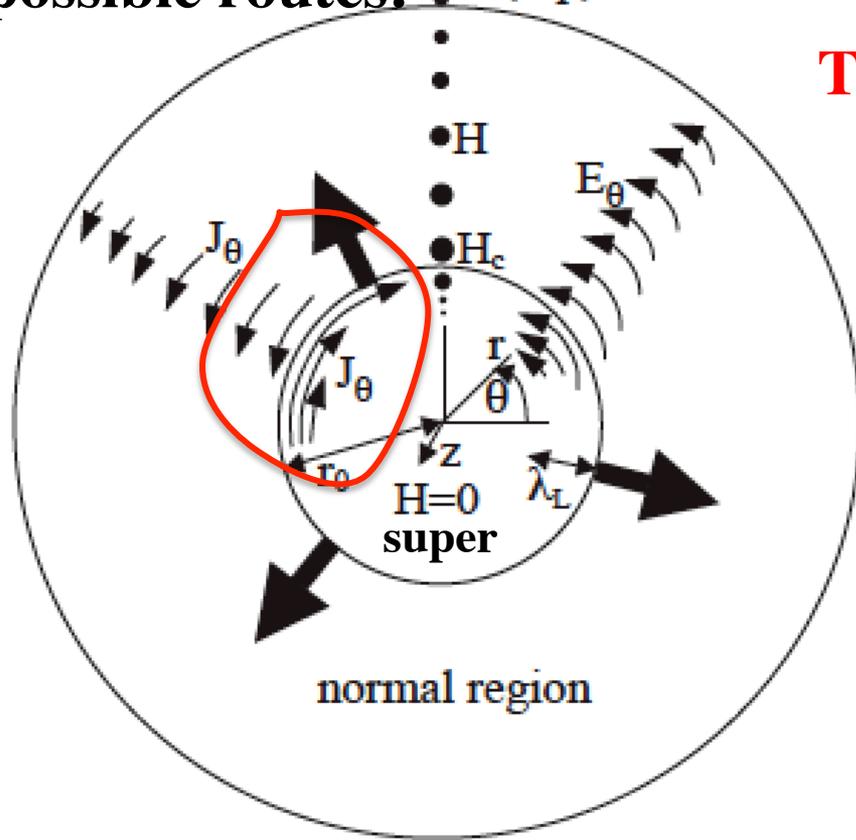
# Meissner effect puzzle

possible routes:

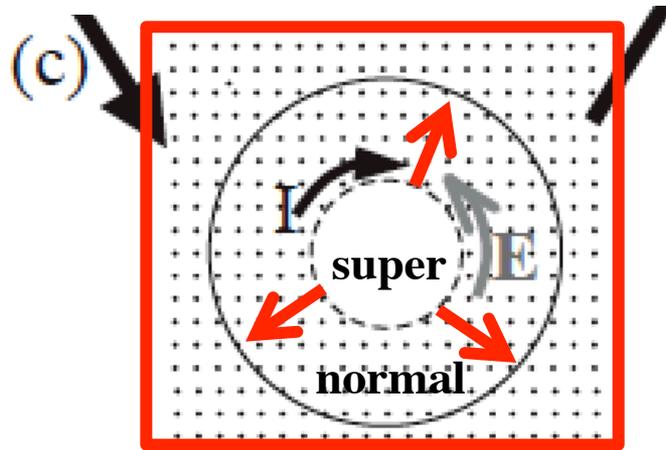


Meissner effect puzzle

possible routes:  $H_c(1-p)$

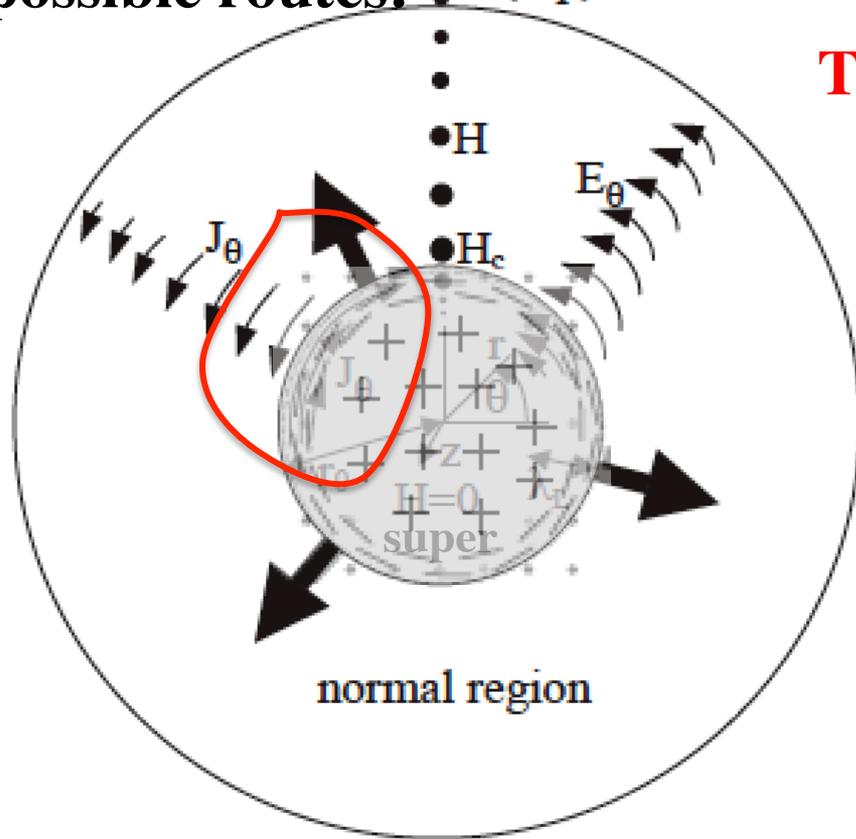


The answer:

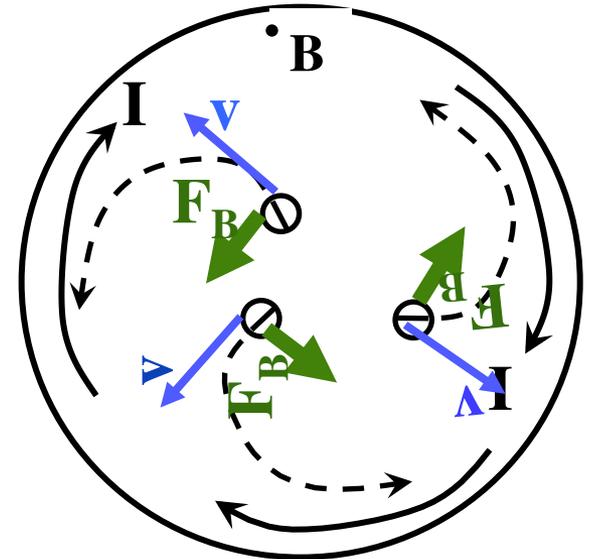


**Meissner effect puzzle**

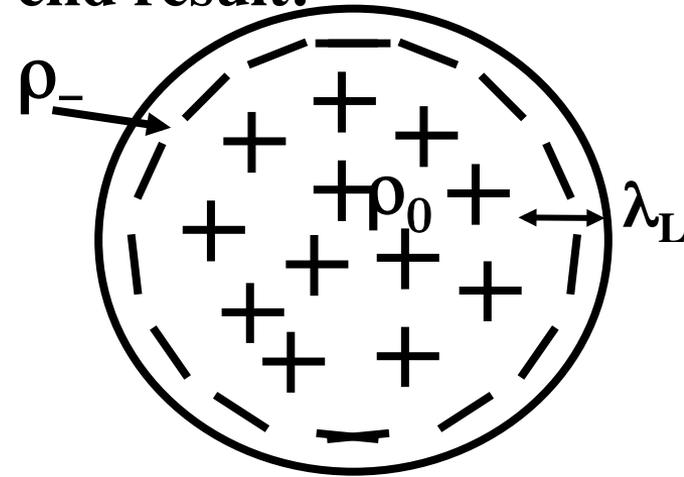
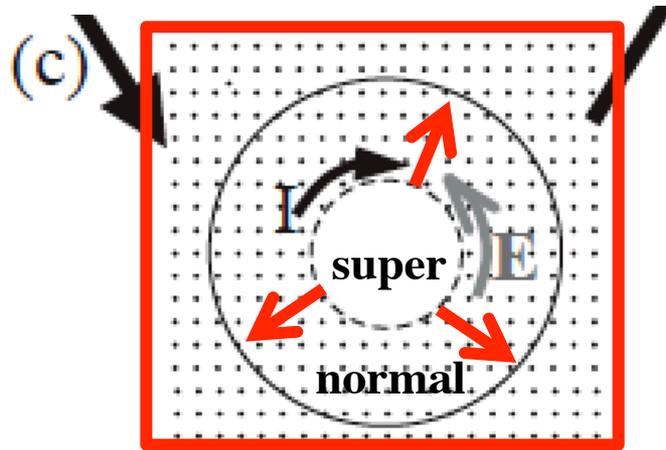
possible routes:  $H_c(1-p)$



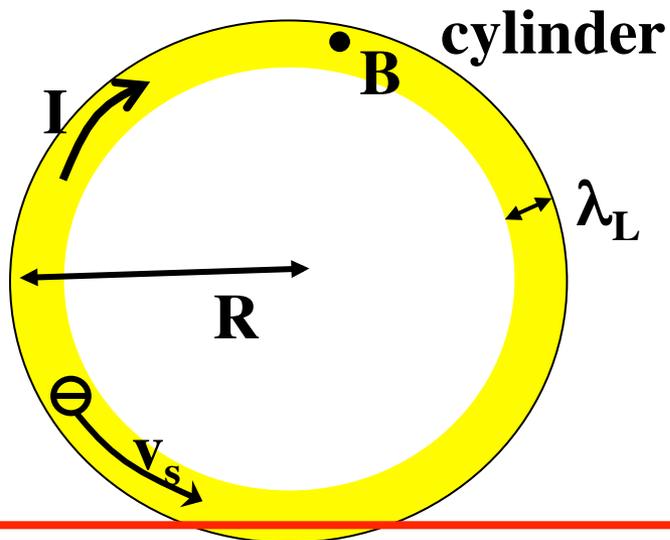
**The answer:**  $\vec{F}_B = \frac{e}{c} \vec{v} \times \vec{B}$  **Lorentz force**



**end result:**



# Dynamics of the Meissner effect



## BCS / London :

$$v_s = \frac{1}{m_e} \left( p - \frac{e}{c} A \right)$$

$$\langle p \rangle = 0 \implies$$

$$v_s = \frac{e}{m_e c} A$$

$$A = \lambda_L B \implies$$

$$v_s = \frac{e \lambda_L}{m_e c} B$$

orbit expansion:

$$k_F^{-1}$$



Electron moving radially out: Lorentz force deflects e<sup>-</sup>

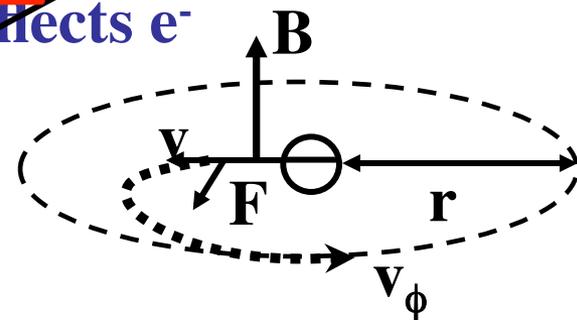
$$\vec{F} = \frac{e}{c} \vec{v} \times \vec{B} + \vec{F}_r = m_e \frac{d\vec{v}}{dt}$$

$$\frac{d}{dt} (\vec{r} \times \vec{v}) = -\frac{e}{2m_e c} (\vec{r} \cdot \vec{v}) \vec{B} = -\frac{e}{2m_e c} \frac{d}{dt} (r^2) \vec{B}$$

$$\implies v_\phi = -\frac{er}{2m_e c} B$$

$$r = 2\lambda_L$$

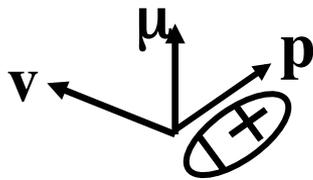
$$\implies v_\phi = \frac{e \lambda_L}{m_e c} B$$



So we learn from the Meissner effect that: transition to superconductivity = expansion of electronic orbit from  $r=k_F^{-1}$  to  $r=2\lambda_L$

What happens when there is no magnetic field?

Spin-orbit force deflects electron in expanding orbit! = “Spin Meissner effect”

$$\vec{p} = \frac{\vec{v}}{c} \times \vec{\mu}$$


moving  $\mu = p =$  electric dipole

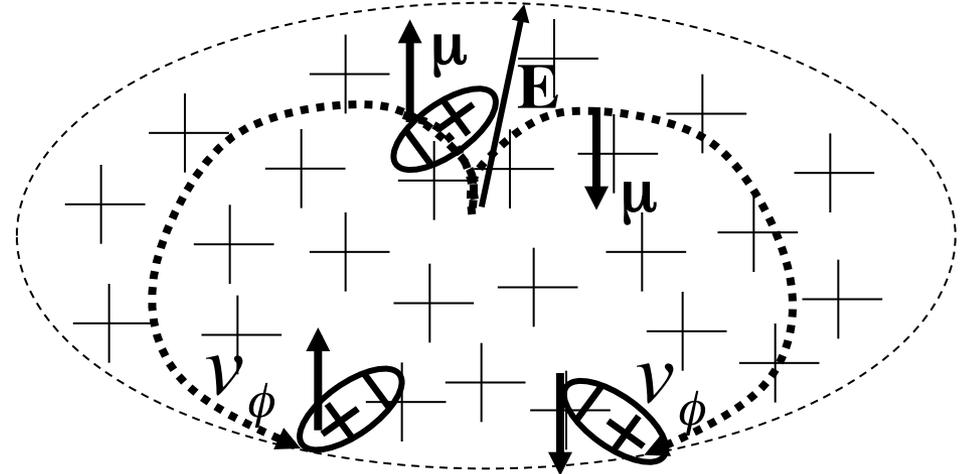
$$\frac{d\vec{L}}{dt} = \vec{\tau} = \vec{p} \times \vec{E} \implies \implies$$

$$m_e \frac{d}{dt} (\vec{r} \times \vec{v}) = \left( \frac{\vec{v}}{c} \times \vec{\mu} \right) \times \vec{E} = \frac{1}{c} (\vec{E} \cdot \vec{v}) \vec{\mu}$$

$$\text{with } \vec{E} = \alpha \vec{r} : m_e \frac{d}{dt} (\vec{r} \times \vec{v}) = \frac{\alpha}{c} (\vec{r} \cdot \vec{v}) \vec{\mu} = \frac{\alpha}{2c} \frac{d}{dt} (r^2) \vec{\mu}$$

$$\implies \vec{r} \times \vec{v} = \frac{\alpha}{2cm_e} (r^2) \vec{\mu} = \frac{Er}{2cm_e} \vec{\mu}$$

$$\implies v_\phi = \frac{E}{2m_e c} \mu_B$$



$$\Rightarrow v_\phi = \frac{E}{2m_e c} \mu_B \quad \text{What's } E?$$

ionic background charge  $\rho$

$$E = 2\pi\rho r, \quad \rho = |e| n_s$$

$$\Rightarrow v_\phi = \frac{2\pi e n_s r}{2m_e c} \mu_B = \frac{\pi e n_s r}{m_e c} \frac{e\hbar}{2m_e c}$$

$$; \text{ with } \frac{4\pi n_s e^2}{m_e c^2} = \frac{1}{\lambda_L^2} \Rightarrow$$

$$v_\phi = \frac{\hbar}{8m_e} \frac{r}{\lambda_L^2}$$

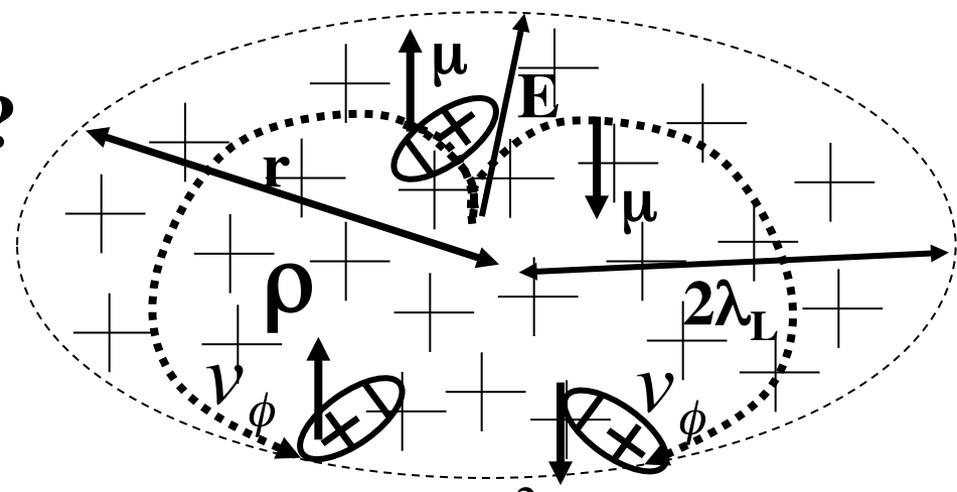
For  $r=2\lambda_L$

$$v_\phi = \frac{\hbar}{4m_e \lambda_L}$$

$$\Rightarrow L = m_e v_\phi r = \dots$$

$$\Rightarrow \vec{r} \times \vec{v} = \frac{\alpha}{2cm_e} (r^2) \vec{\mu} = \frac{Er}{2cm_e} \vec{\mu}$$

$$\Rightarrow v_\phi = \frac{E}{2m_e c} \mu_B$$



$$\Rightarrow v_\phi = \frac{E}{2m_e c} \mu_B \quad \text{What's } E?$$

ionic background charge  $\rho$

$$E = 2\pi\rho r, \quad \rho = |e| n_s$$

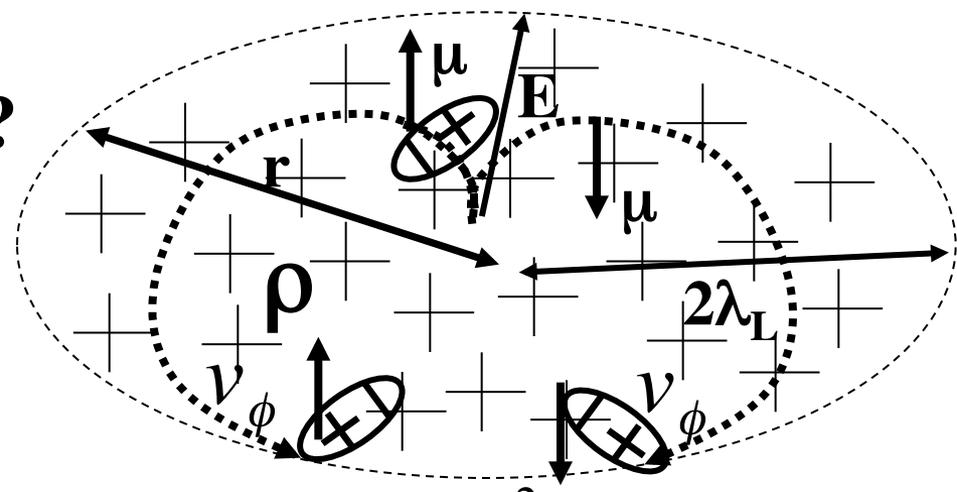
$$\Rightarrow v_\phi = \frac{2\pi e n_s r}{2m_e c} \mu_B = \frac{\pi e n_s r}{m_e c} \frac{e\hbar}{2m_e c} \quad ; \text{ with } \frac{4\pi n_s e^2}{m_e c^2} = \frac{1}{\lambda_T^2} \Rightarrow$$

$$v_\phi = \frac{\hbar}{8m_e} \frac{r}{\lambda_L^2}$$

For  $r = 2\lambda_L$

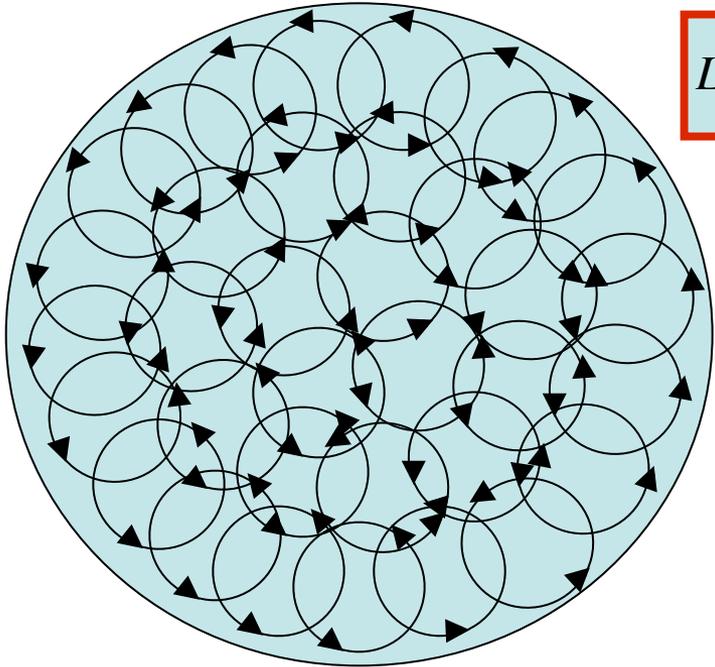
$$\Rightarrow L = m_e v_\phi r = \dots = \frac{\hbar}{2} \text{ !!!!!!!!!}$$

$$\Rightarrow \vec{r} \times \vec{v} = \frac{\alpha}{2cm_e} (r^2) \vec{\mu} = \frac{Er}{2c} \vec{u}$$



# Ground state of a superconductor (no magnetic field applied)

$r=2\lambda_L$  orbits

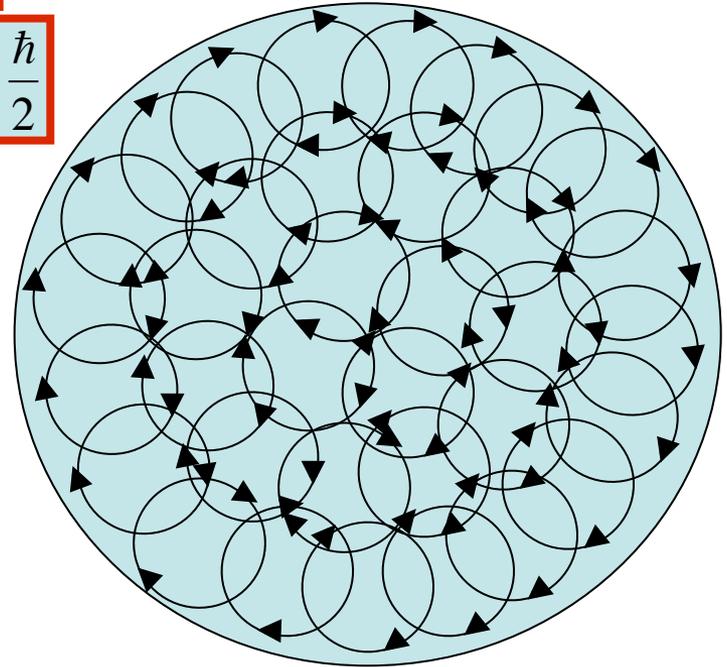


Electron spin into screen

$$v_\phi = \frac{\hbar}{4m_e\lambda_L}$$

$$L = m_e v_\phi \cdot (2\lambda_L) = \frac{\hbar}{2}$$

$r=2\lambda_L$  orbits



Electron spin out of screen

**Macroscopic zero point motion in the ground state of superconductors**

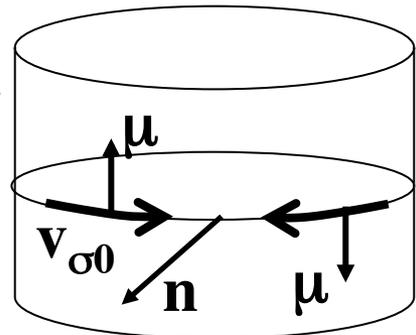
Currents in the interior cancel out, near the surface survive  
 $\implies$  there is a **spontaneous spin current** in the ground state of superconductors near the surface!

*There is a spontaneous spin current in the ground state of superconductors, flowing within  $\lambda_L$  of the surface*

$$\vec{v}_{\sigma 0} = -\frac{\hbar}{4m_e\lambda_L} \vec{\sigma} \times \hat{n}$$

*no external fields applied*

$$\vec{\mu} = \frac{e\hbar}{2m_e c} \vec{\sigma}$$

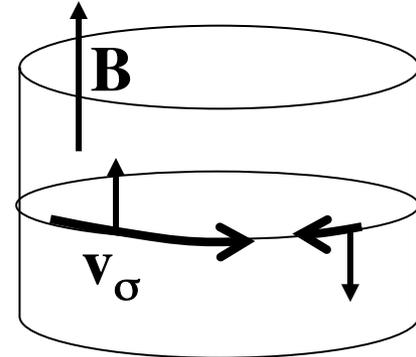


For  $\lambda_L=400\text{\AA}$ ,  $v_{\sigma 0}=72,395\text{cm/s}$

# of carriers in the spin current:  $n_s$

When a magnetic field is applied:

$$\vec{v}_{\sigma} = \vec{v}_{\sigma 0} - \frac{e}{m_e c} \lambda_L \vec{B} \times \hat{n}$$



The slowed-down spin component stops when

$$B = \frac{m_e c}{e\lambda_L} v_{\sigma 0} = \frac{\hbar c}{4e\lambda_L^2} = \frac{\phi_0}{4\pi\lambda_L^2} \sim H_{c1}!$$

Electronic orbits have  $2\lambda_L$  radius (to explain Meissner effect)

Angular momentum:  $L = m_e v_{\sigma 0} (2\lambda_L) \implies L = \hbar/2$

**Microscopic derivation:** (J. Sup.Nov.Mag, 26, 2239 (2013))

$$H_{spin-orbit} = -\frac{e\hbar}{4m_e^2c^2} \vec{\sigma} \cdot (\vec{E} \times \vec{p}) \quad (\text{from Dirac eq.; } E=\text{electric field})$$

**Single particle Hamiltonian:** (Aharonov-Casher) **Electrostatic energy of charge expulsion**

$$H = \frac{1}{2m_e} (\vec{p} - \frac{e}{c} \vec{A}_\sigma)^2 = \frac{p^2}{2m_e} - \frac{e}{m_e c} \vec{A}_\sigma \cdot \vec{p} + \frac{e^2}{2m_e c^2} A_\sigma^2$$

$$\vec{A}_\sigma = \frac{\hbar}{4m_e c} \vec{\sigma} \times \vec{E}$$

$$\vec{E} = 2\pi |e| n_s \vec{r} = \frac{m_e c^2}{2e\lambda_L^2} \vec{r}$$

$$\implies \vec{A}_\sigma = -\frac{\hbar c}{8e\lambda_L^2} \vec{\sigma} \times \vec{r} \equiv E_m \frac{\vec{\sigma} \times \vec{r}}{2} \equiv \frac{\vec{B}_\sigma \times \vec{r}}{2}$$

$$E_m = -\frac{\hbar c}{4e\lambda_L^2} ; \quad \vec{B}_\sigma = E_m \vec{\sigma}$$

**'Magnetic length':**  $\ell_{B_\sigma} = \left(\frac{\hbar c}{|e| B_\sigma}\right)^{1/2} = 2\lambda_L$

$$H = \frac{p^2}{2m_e} + \frac{\hbar q_0}{2m_e} (\vec{\sigma} \times \hat{n}) \cdot \vec{p} + \frac{\hbar^2 q_0^2}{8m_e}$$

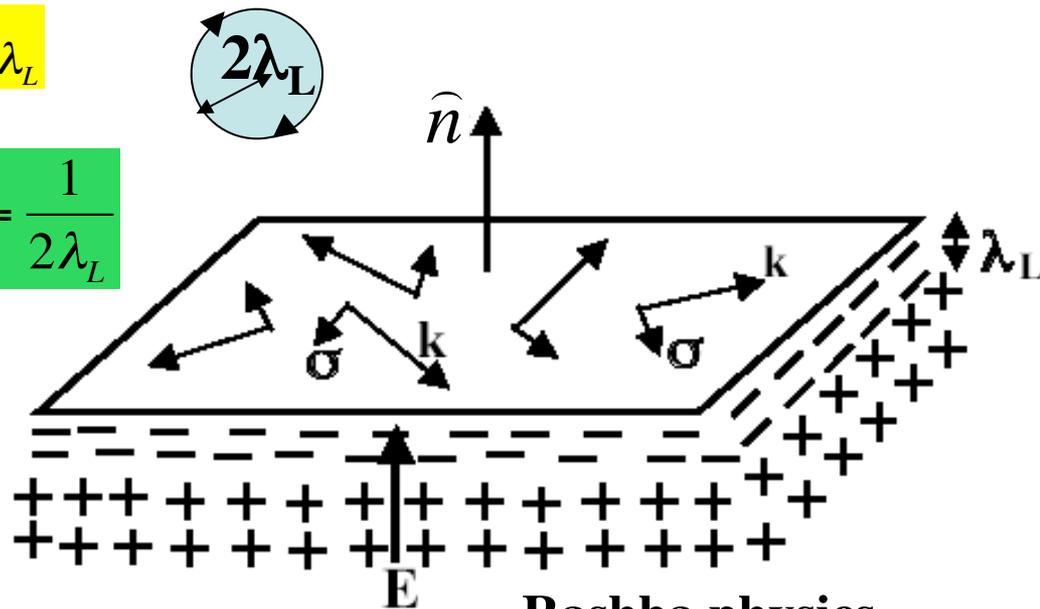
$$q_0 = \frac{1}{2\lambda_L}$$

$$\varepsilon_{k\sigma} = \frac{\hbar^2 k^2}{2m_e} - \frac{\hbar^2}{2m_e} q_0 \vec{k} \cdot (\vec{\sigma} \times \hat{n}) + \frac{\hbar^2 q_0^2}{8m_e}$$

$$\vec{v}_{k\sigma} = \frac{1}{\hbar} \frac{\partial \varepsilon_{k\sigma}}{\partial \vec{k}} = \frac{\hbar \vec{k}}{m_e} - \frac{\hbar q_0}{2m_e} (\vec{\sigma} \times \hat{n})$$

$$\vec{v}_{\sigma 0} = -\frac{\hbar}{4m_e \lambda_L} \vec{\sigma} \times \hat{n}$$

$$\varepsilon_{k\sigma} = \frac{\hbar^2}{2m_e} \left(k \mp \frac{q_0}{2}\right)^2$$

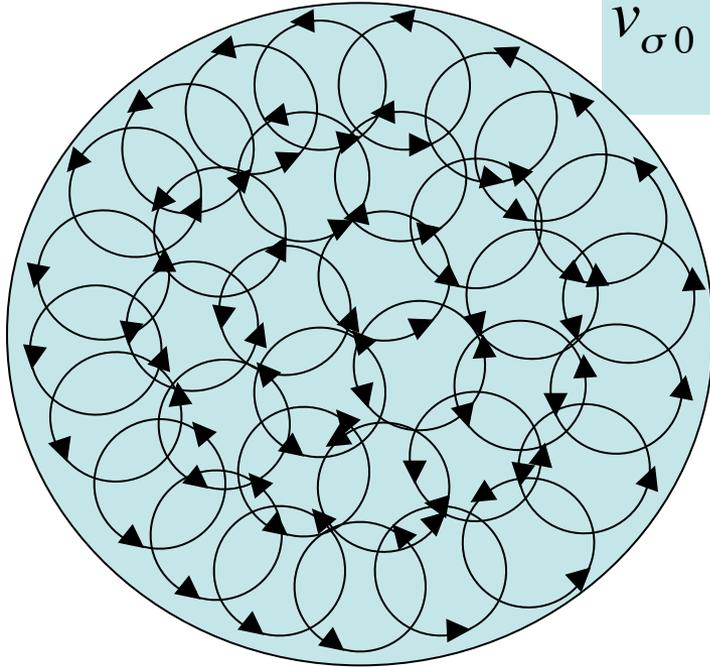


**Rashba physics**

We now have 2 new pieces of physics of superconductors:

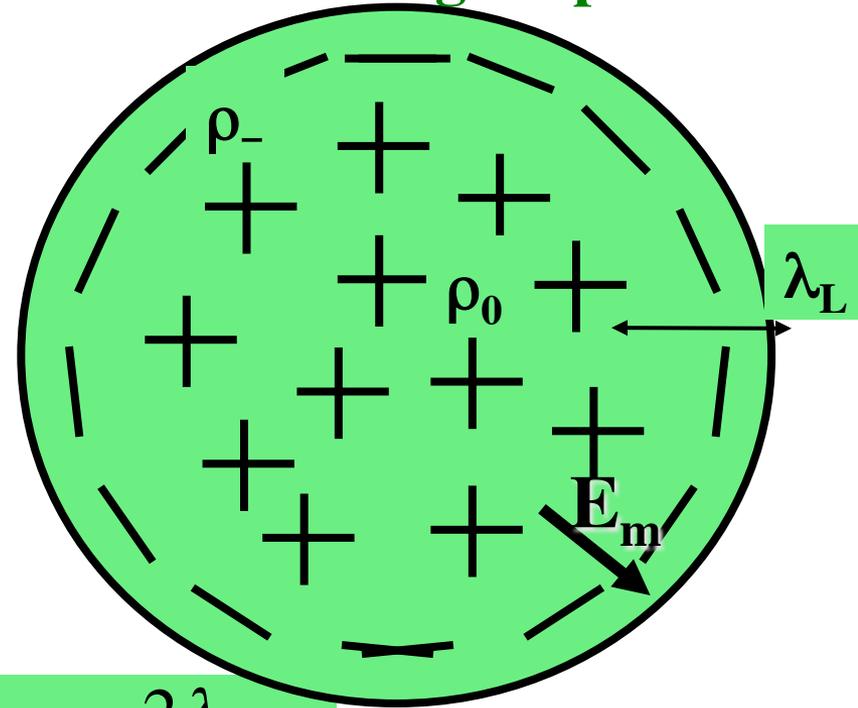
$r=2\lambda_L$  orbits

$$v_{\sigma 0} = \frac{\hbar}{4m_e \lambda_L} \sigma$$



spin current

charge expulsion



$$\rho_0 = -\frac{2\lambda_L}{R} \rho_-$$

$$E_m = -4\pi\lambda_L \rho_-$$

*What is  $E_m$ ?*

**How are they related?**

**How much charge is expelled?**

# Spin current electrodynamics

$$J_\sigma(\vec{r}, t) = (\vec{J}_\sigma(\vec{r}, t), ic\rho_\sigma(\vec{r}, t))$$

$$J_{\sigma 0} = (\vec{J}_{\sigma 0}, ic\rho_{\sigma 0})$$

$$A_\sigma(\vec{r}, t) = (\vec{A}_\sigma(\vec{r}, t), i\phi_\sigma(\vec{r}, t))$$

$$A_{\sigma 0}(\vec{r}) = (\vec{A}_{\sigma 0}(\vec{r}), i\phi_{\sigma 0}(\vec{r}))$$

$$J_\sigma(\vec{r}, t) - J_{\sigma 0} = -\frac{c}{8\pi\lambda_L^2} (A_\sigma(\vec{r}, t) - A_{\sigma 0}(\vec{r}))$$

$$\vec{A}_\sigma(\vec{r}, t) = \lambda_L \vec{\sigma} \times \vec{E}(\vec{r}, t) + \vec{A}(\vec{r}, t)$$

$$\vec{A}_{\sigma 0}(\vec{r}) = \lambda_L \vec{\sigma} \times \vec{E}_0(\vec{r})$$

$$\phi_\sigma(\vec{r}, t) = -\lambda_L \vec{\sigma} \cdot B(\vec{r}, t) + \phi(\vec{r}, t)$$

$$\phi_{\sigma 0}(\vec{r}) = \phi_0(\vec{r})$$

$$(A_\sigma)_\alpha = \frac{i\lambda_L}{2} \epsilon_{\alpha\beta\gamma\delta} \sigma_\beta F_{\gamma\delta} + A_\alpha$$

Ann. der Phys.17, 380 (2008)

$$J_\sigma(\vec{r}, t) = \left( \frac{en_s}{2} \vec{v}_\sigma(\vec{r}, t), ic\rho_\sigma(\vec{r}, t) \right)$$

$$J_{\sigma 0} = \left( \frac{en_s}{2} \vec{v}_{\sigma 0}, ic\rho_{\sigma 0} \right)$$

$$J_\sigma(\vec{r}, t) = \rho_\sigma(\vec{r}, t) c (-\vec{\sigma} \times \hat{r}, i)$$

$$J_{\sigma 0} = \frac{\rho_0 c}{2} (-\vec{\sigma} \times \hat{r}, i)$$

$$\square^2 (A_\sigma - A_{\sigma 0}) = \frac{1}{\lambda_L^2} (A_\sigma - A_{\sigma 0})$$

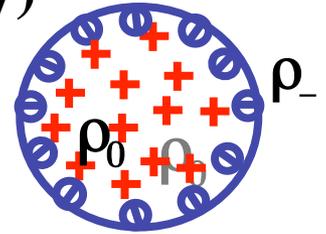
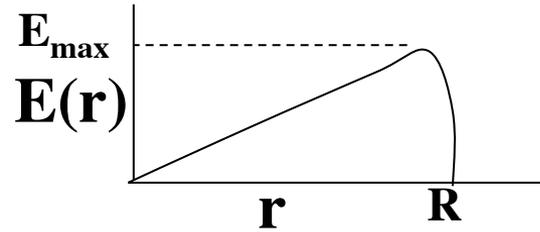
$$\square^2 (J_\sigma - J_{\sigma 0}) = \frac{1}{\lambda_L^2} (J_\sigma - J_{\sigma 0}).$$

$$\square^2 = \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}$$

$$E_m = -\frac{\hbar c}{4e\lambda_L^2} = \frac{\phi_0}{4\pi\lambda_L^2} \sim H_{c1}$$

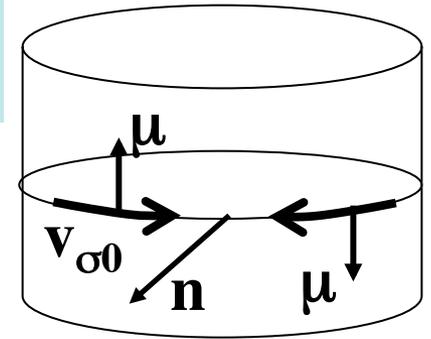
$$E_m = -4\pi\lambda_L\rho_- \quad (\text{charge neutrality})$$

$$\rho_- = n_s e \frac{v_{\sigma 0}}{c} = \frac{\hbar c}{16\pi e\lambda_L^3}$$



$$\vec{\nabla} \times J_\sigma(\vec{r}) = -\frac{c}{2\lambda_L}(\rho(\vec{r}) - \rho_0)$$

$$v_{\sigma 0} = \frac{\hbar}{4m_e\lambda_L} \sigma$$



$$J_\sigma(\vec{r}) \equiv en_s v_\sigma(\vec{r}) = -\frac{c}{8\pi\lambda_L} \vec{\sigma} \times (\vec{E}(\vec{r}) - \vec{E}_0(\vec{r}))$$

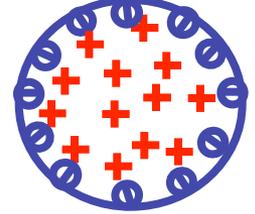
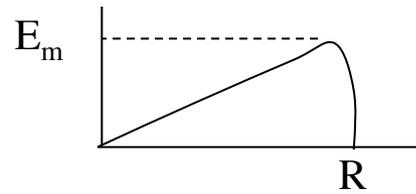
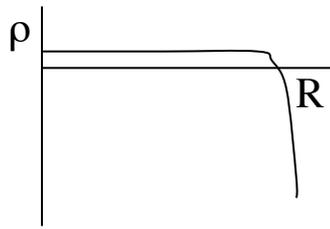
$$n_s \left( \frac{1}{2} m_e v_{\sigma 0}^2 \right) = \frac{E_m^2}{8\pi}$$

(Recall  $n_s \left( \frac{1}{2} m_e v_s^2 \right) = \frac{B^2}{8\pi}$ )

**kin. energy of spin current  
=electrostatic energy**

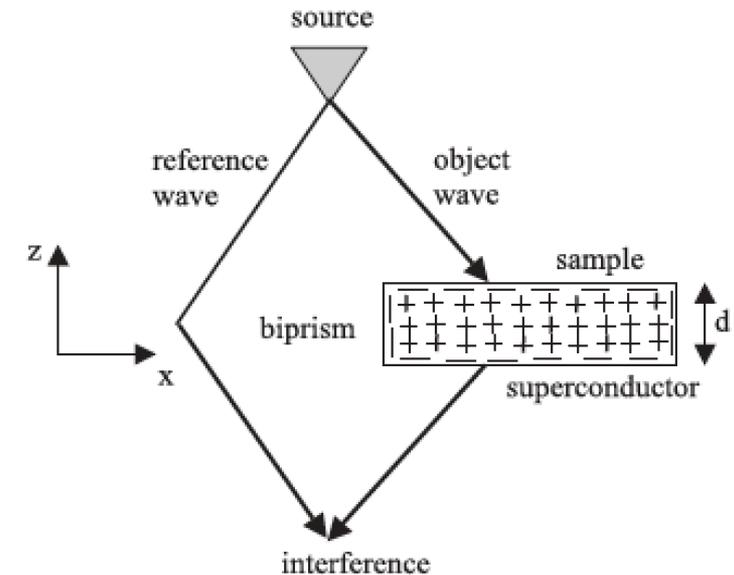
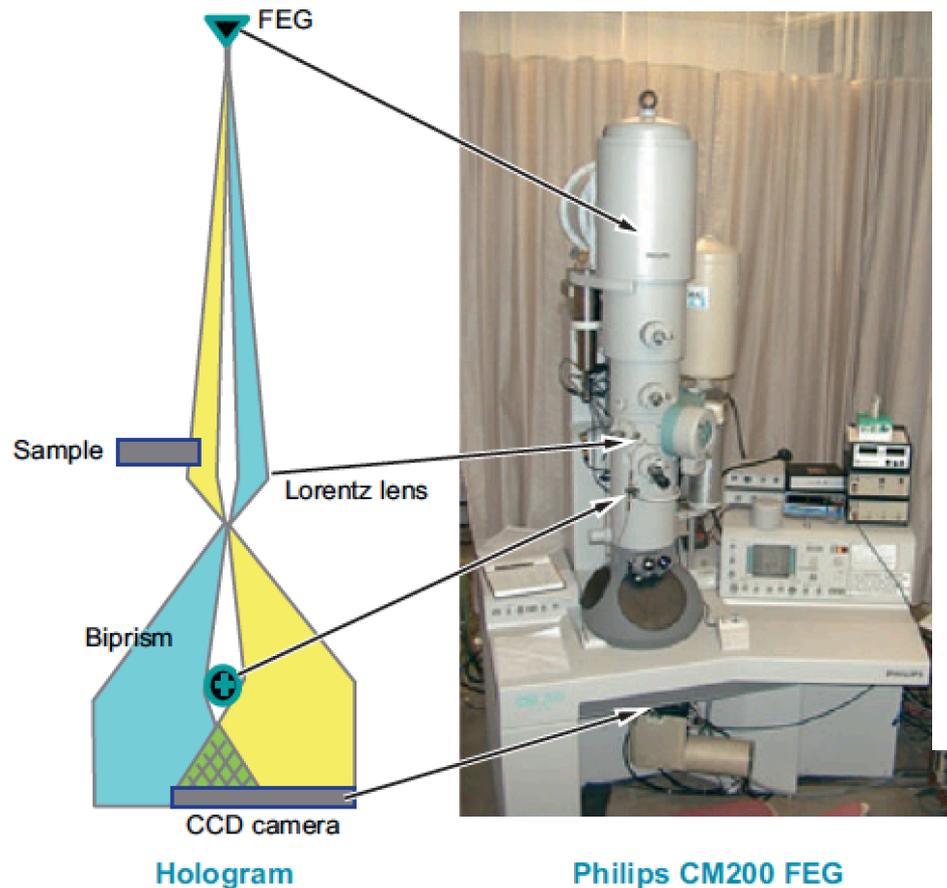
**kin. energy of charge current  
=magnetostatic energy**

# How much charge is expelled?



| element | $T_c$ (K) | $H_c$ (G) | $\lambda_L$ (Å) | Extra electrons | $E_m$<br>(Volts/cm) |
|---------|-----------|-----------|-----------------|-----------------|---------------------|
| Al      | 1.14      | 105       | 500             | 1/17 mill       | 31,500              |
| Sn      | 3.72      | 309       | 510             | 1/3.7 mill      | 92,700              |
| Hg      | 4.15      | 412       | 410             | 1/2.5 mill      | 123,600             |
| Pb      | 7.19      | 803       | 390             | 1/1 mill        | 240,900             |
| Nb      | 9.50      | 1980      | 400             | 1/1.3 mill      | 308,400             |

# Electron holography: measure mean inner potential in normal and superconducting state



**Fig. 1.** Schematic depiction of off-axis electron holography experiment. The wave traversing the superconducting sample will advance its phase due to the presence of an additional positive potential in the superconductor.

phase shift  $\phi(x) = C_E \int V(x, z) dz$

$$C_E = \frac{2\pi|e|}{\lambda E} \frac{E + m_e c^2}{E + 2m_e c^2}$$

$\phi = C_E \bar{V}_0 d$  normal metal  
d=thickness

**Figure 1**

(Left) Schematic illustration showing the typical configuration used for off-axis electron holography in the transmission electron microscope. Essential components are the field-emission gun (FEG) electron source used to provide coherent incident illumination and the electrostatic biprism, which causes overlap of object and (vacuum) reference waves. (Right) Photograph of a Philips CM200 transmission electron microscope equipped with an FEG electron source, Lorentz minilens beneath the normal objective lens to provide field-free imaging for magnetic materials, electrostatic biprism, and charge-coupled-device (CCD) camera for the quantitative recording of holograms.

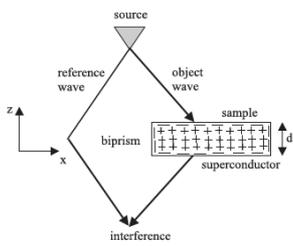


Fig. 1. Schematic depiction of off-axis electron holography experiment. The wave traversing the superconducting sample will advance its phase due presence of an additional positive potential in the superconductor.

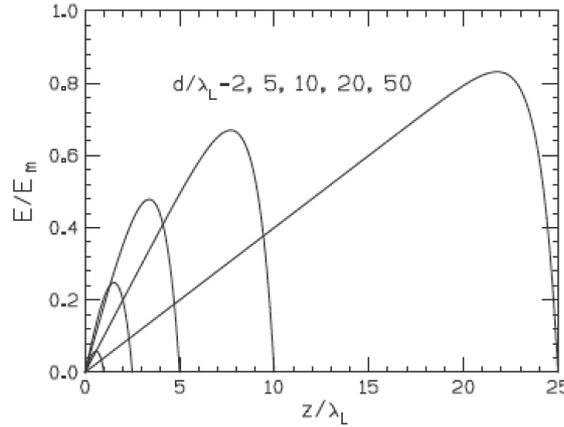


Fig. 2. Electric field resulting from charge expulsion from the center of the ( $z = 0$ ) to the upper edge ( $z = d/2$ ) for samples of various thicknesses  $d$ , far from lateral edges of the sample. The electric field points in the  $+z$  direction.

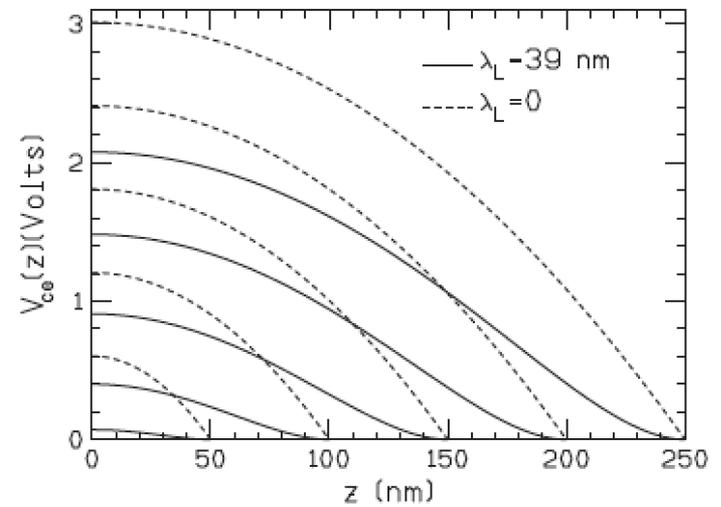


Fig. 3. Electric potential resulting from charge expulsion for  $z$  ranging from the center of the sample ( $z = 0$ ) to the upper edge ( $z = d/2$ ) for samples of thicknesses  $d = 100, 200, 300, 400, 500$  nm, far from the lateral edges of the sample, for  $\lambda_L = 39$  nm (solid lines) and  $\lambda_L = 0$  (dashed lines). The electric potential goes to zero at the upper edge of the sample ( $z = d/2$ ). The magnitude of the electric potential

$$V_{ce}(z) = \frac{E_m d}{4} \left( 1 - \frac{4z^2}{d^2} \right) + \frac{E_m \lambda_L}{\sinh\left(\frac{d}{2\lambda_L}\right)} \left[ \cosh\left(\frac{z}{\lambda_L}\right) - \cosh\left(\frac{d}{2\lambda_L}\right) \right]$$

$$\bar{V}_{ce} = \frac{2}{d} \int_0^{d/2} V_{ce}(z) dz$$

**d=thickness**

$$\phi = C_E (V_0 + \bar{V}_{ce}) \times d$$

$$\bar{V}_{ce} = \frac{E_m d}{6} + 2E_m \frac{\lambda_L^2}{d} - E_m \lambda_L \frac{\cosh\left(\frac{d}{2\lambda_L}\right)}{\sinh\left(\frac{d}{2\lambda_L}\right)}$$

$$\phi = C_E \left( \bar{V}_0 d + \frac{E_m}{6} d^2 \right)$$

$$T < 0.10 T_c$$

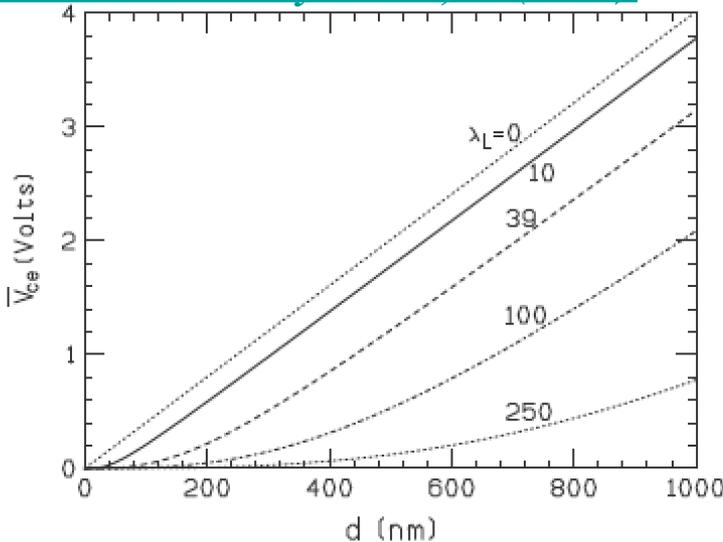


Fig. 4. Mean inner potential resulting from charge expulsion as a function of sample thickness  $d$  for various values of the London penetration depth (numbers next to the lines, in nm). The value of the maximum electric field corresponds to the case of Pb, and  $\lambda_L = 39$  nm corresponds to Pb. The total mean inner potential has in addition a thickness-independent contribution which is the same as in the normal state.

**JEH, [Ultramicroscopy 133, 67 \(2013\)](#);**

**[Physica C 490, 1 \(2013\)](#);**

**[Annalen der Physik 526, 63 \(2014\)](#)**

# Tunneling asymmetry prediction (1989)

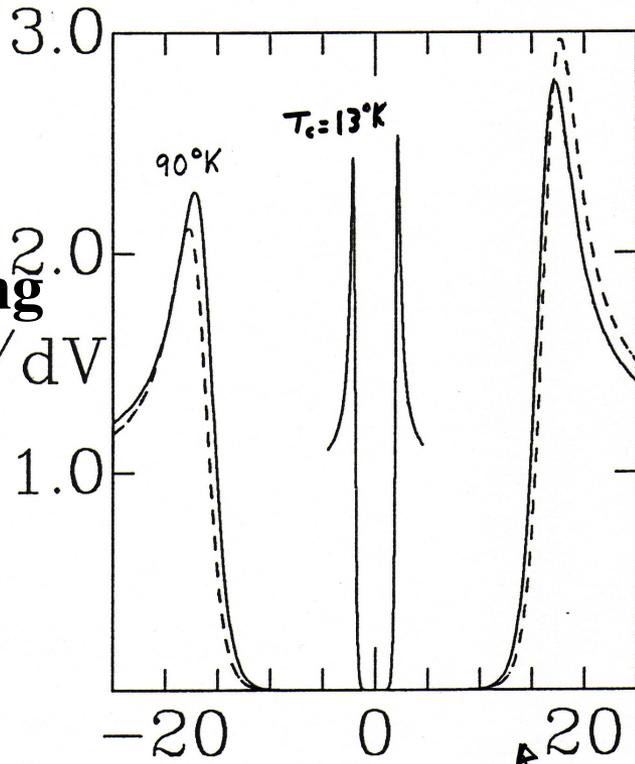
Physica C 159 (1989) 157-160

F. MARSIGLIO and J.E. HIRSCH

## TUNNELING ASYMMETRY: A TEST OF SUPERCONDUCTIVITY MECHANISMS

**N-I-S  
tunneling**

$dI/dV$



$U = 5e$   
 $V = 0$   
 $\Delta t \sim 0$   
 $T/T_c =$

(1989)

(universal sign)

$$\frac{N_s(\omega)}{N_{normal}} = \frac{1}{a} \left[ \Theta(\omega - \mu - \Delta_0) \frac{\omega - \mu + V}{\sqrt{(\omega - \mu)^2 - \Delta_0^2}} + \Theta(-(\omega - \mu) - \Delta_0) \frac{\mu - \omega - V}{\sqrt{(\omega - \mu)^2 - \Delta_0^2}} \right]$$

Gap function has slope  
of universal sign

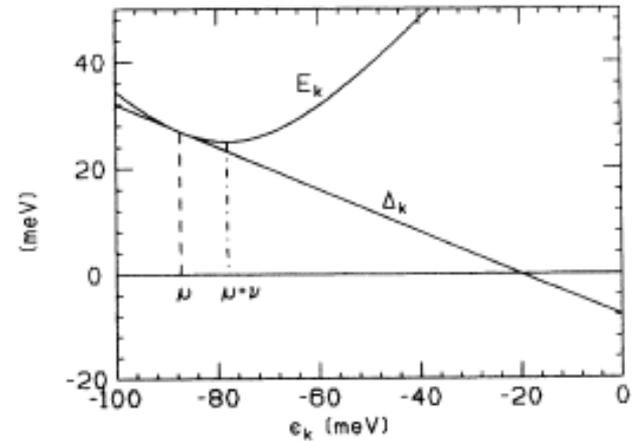


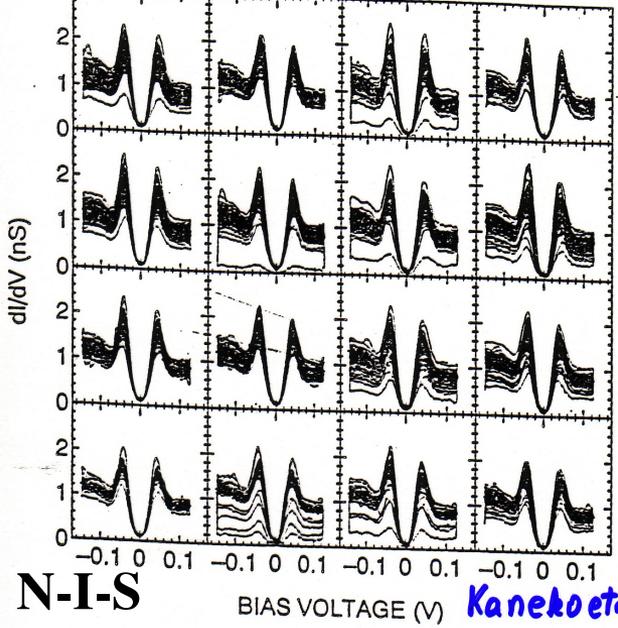
FIG. 1. Energy gap function  $\Delta_k$  and quasiparticle energy  $E_k$  versus hole kinetic energy  $\epsilon_k$  in the model of hole superconductivity (schematic). Only the lower half of the hole band (upper half of the electron band) is shown. Note that the minimum in the quasiparticle energy is shifted from the chemical potential  $\mu$  to  $\mu + \nu$ .

(1989)

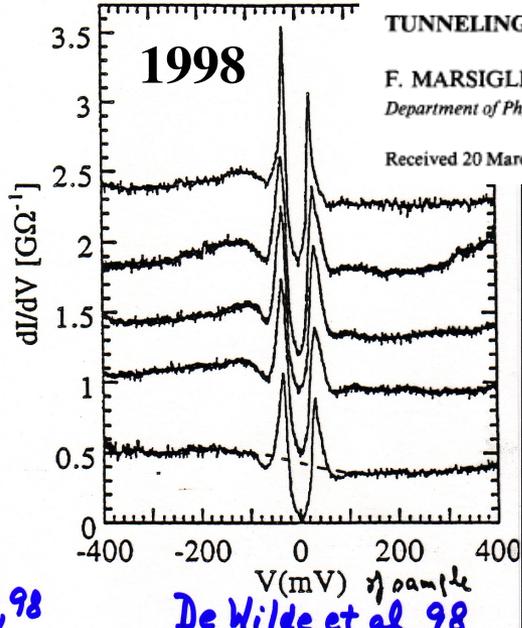
F. MARSIGLIO and J.E. HIRSCH

Department of Physics, B-019, University of California, San Diego, La Jolla, CA 92093, USA

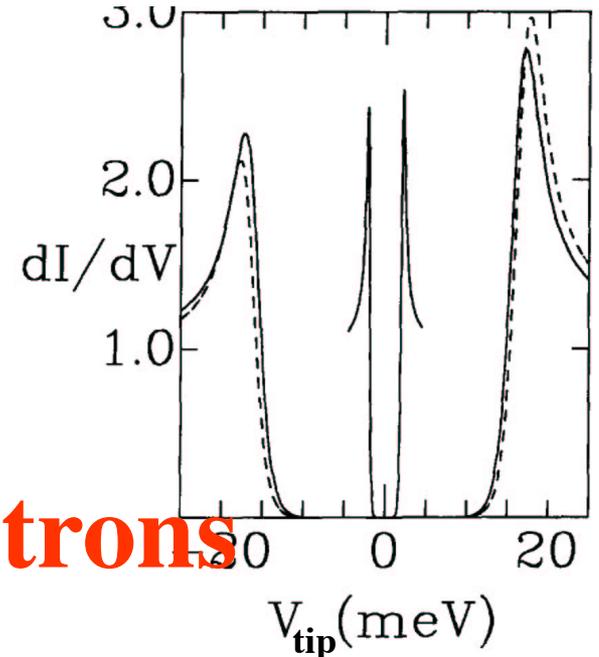
Received 20 March 1989



**N-I-S**  
BIAS VOLTAGE (V) *Kaneko et al., 98*



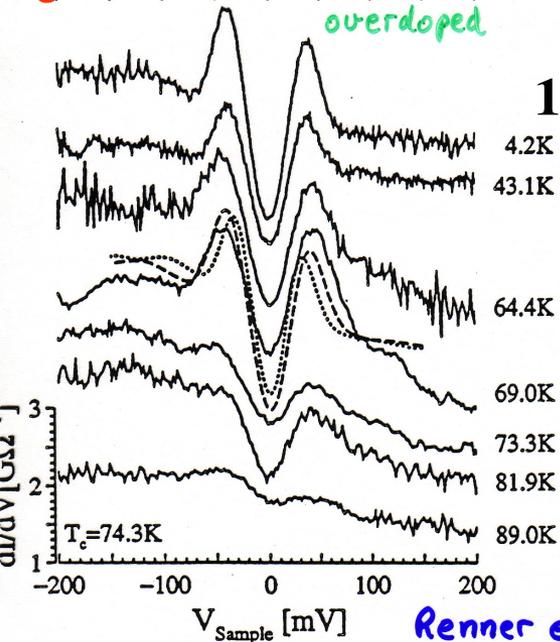
1998  
V(mV) of sample  
*De Wilde et al., 98*



$V_{tip}$  (meV)

current is larger for negatively biased sample

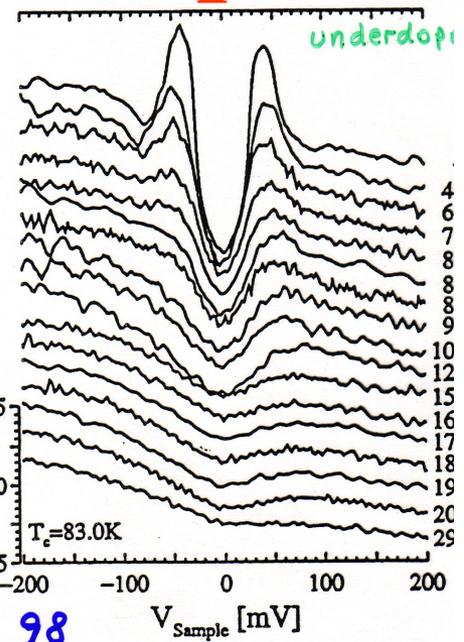
system wants to expel electrons



*overdoped*

1998

*Renner et al., 98*



*underdoped*

$T_c = 83.0K$

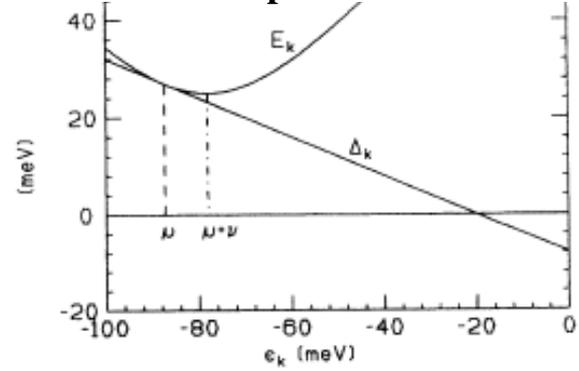


FIG. 1. Energy gap function  $\Delta_k$  and quasiparticle energy  $E_k$  versus hole kinetic energy  $\epsilon_k$  in the model of hole superconductivity (schematic). Only the lower half of the hole band (upper half of the electron band) is shown. Note that the minimum in the quasiparticle energy is shifted from the chemical potential  $\mu$  to  $\mu + \nu$ .

Theory of asymmetric tunneling in the cuprate superconductors

P.W. Anderson <sup>\*</sup>, N.P. Ong

2006

Competing order and the asymmetric tunneling spectrum  
in high temperature cuprate superconductors

Jiang-Ping Hu and Kangjun Seo

2006

*for hole-  
doped  
cuprates*

On the origin of the tunnelling asymmetry in the  
cuprate superconductors: a variational perspective

Hong-Yu Yang<sup>1</sup>, Fan Yang<sup>2</sup>, Yong-Jin Jiang<sup>3</sup> and Tao Li<sup>4</sup>

2007

Origin of the Electron-Hole Asymmetry in the Scanning Tunneling Spectrum  
of the High-Temperature  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  Superconductor

Jouko Nieminen,<sup>1,2,\*</sup> Hsin Lin,<sup>2</sup> R. S. Markiewicz,<sup>2</sup> and A. Bansil<sup>2</sup>

2009

Dynamical Particle-Hole Asymmetry in High-Temperature Cuprate Superconductors

B. Sriram Shastry

2012

Pseudogap-induced asymmetric tunneling in cuprate superconductors

Lülin Kuang<sup>a</sup>, Huaisong Zhao<sup>b</sup>, Shiping Feng<sup>a,\*</sup>

2014

# TUNNELING ASYMMETRY: A TEST OF SUPERCONDUCTIVITY MECHANISMS

F. MARSIGLIO and J.E. HIRSCH

1989

*Department of Physics, B-019, University of California, San Diego, La Jolla, CA 92093, USA*

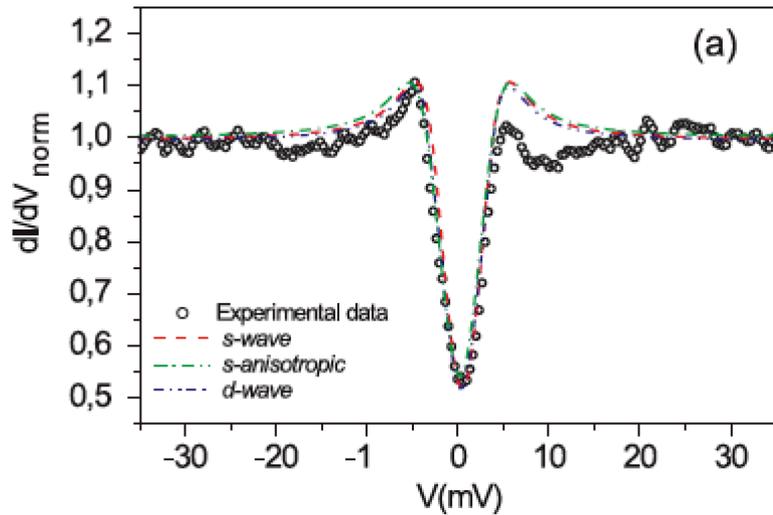
Received 20 March 1989

Within the conventional electron-phonon mechanism of superconductivity, normal-insulator-superconductor (N-I-S) tunneling characteristics are expected to be symmetric with respect to the sign of the bias voltage. A recently proposed mechanism of superconductivity based on pairing of hole carriers predicts an asymmetry of universal sign: the tunneling current should be larger for a negatively biased sample. We suggest a search for this asymmetry in conventional superconductors, as well as in "hole" and "electron" oxide superconductors: its systematic observation would provide direct evidence in favor of the hole pairing mechanism of superconductivity.

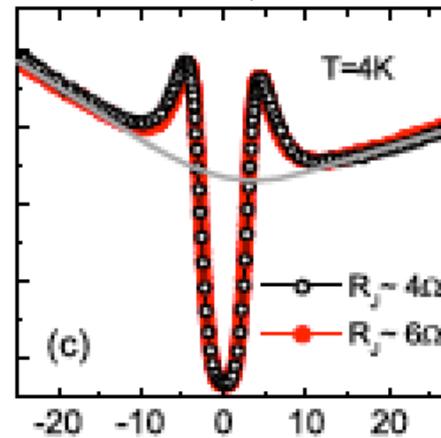
asymmetry despite the low  $T_c$ . Finally, we predict that this effect, *with the same sign*, will be observed in the recently discovered "electron-carrier" oxide super-

# N-I-S tunneling in **electron-doped cuprates** is also asymmetric with asymmetry of the same sign

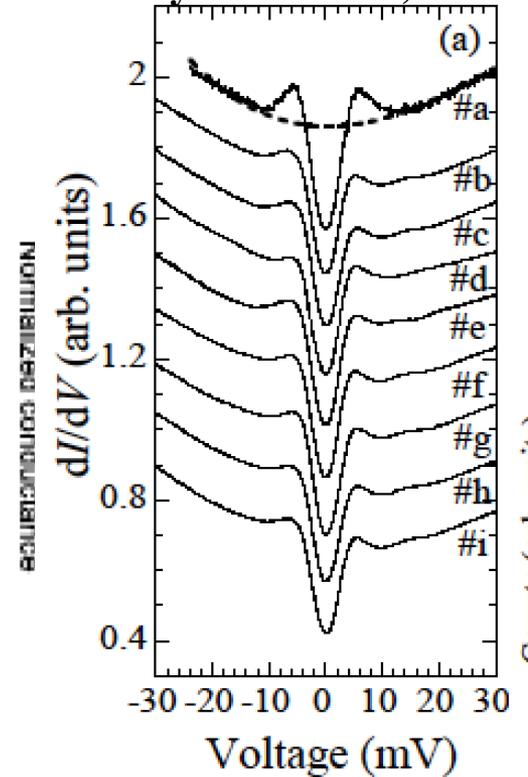
J. Phys.: Condens. Matter 22 (2010) 045702 **Giubileo et al, 2010**



**Shan et al, 2008**

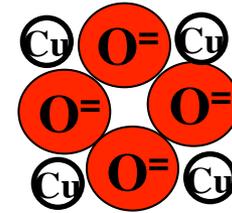


**Miyakawa et al, 2009**

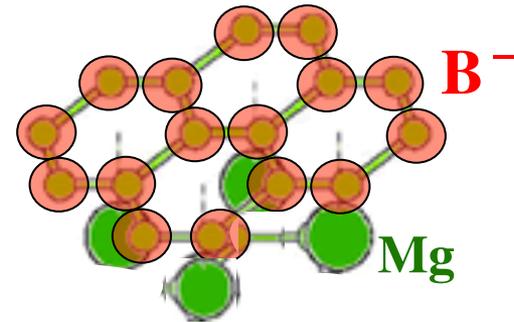


**The three (so far) ways to reach high  $T_c$ : (2011)**  
= three ways to pack **big negative ions** very close together, and have **holes** conducting through them:

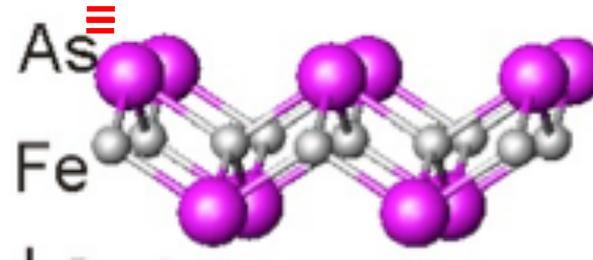
1) Coplanar cation-anion  
(cuprates)



2) Planes of anions only  
( $\text{MgB}_2$ )



3) Cation-anion tetrahedra  
( $\text{FeAs}$ ,  $\text{FeSe}$ , ...)

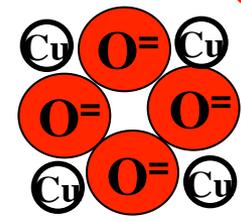


Cations should be small

# The three (so far) ways to reach high $T_c$ : (2011)

= three ways to pack **big negative ions** very close together, and have holes conducting through them:

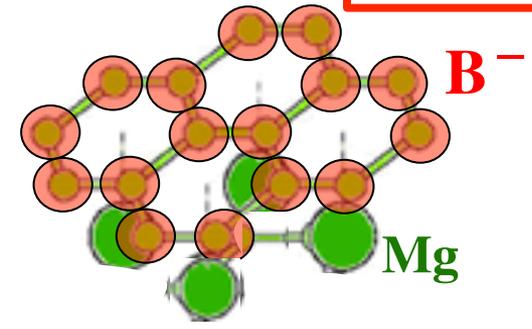
1) Coplanar cation-anion  
(cuprates)



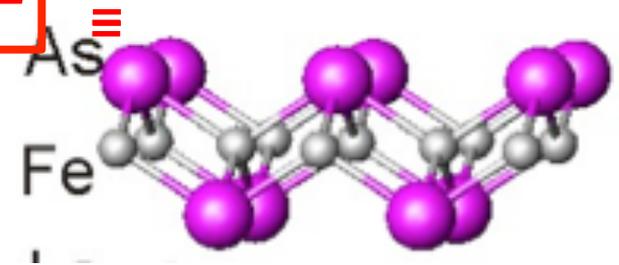
*how about very high pressure?*

2) Planes of anions only  
(MgB<sub>2</sub>)

*how about S=?  
r=1.84Å, vs  
r=1.40Å for O=*



3) Cation-anion tetrahedra  
(FeAs, FeSe, ...)



Cations should be small

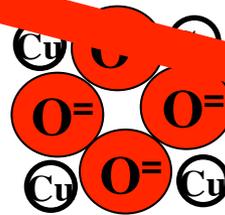
*how about hydrogen?*

# Conventional superconductivity at 203 kelvin at high pressures in the sulfur hydride system

A. P. Drozdov<sup>1\*</sup>, M. I. Erements<sup>1\*</sup>, I. A. Troyan<sup>1</sup>, V. Ksenofontov<sup>2</sup> & S. I. Shylin<sup>2</sup>

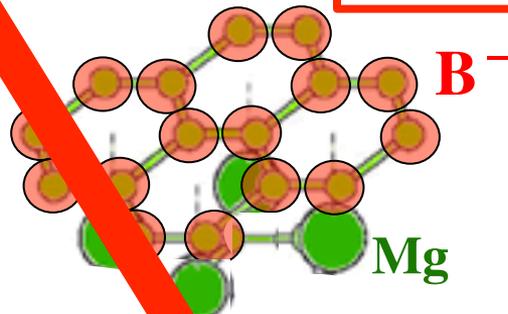
Nature, 2015 <sup>ve</sup>

1) Coplanar cation-anion  
(cuprates)



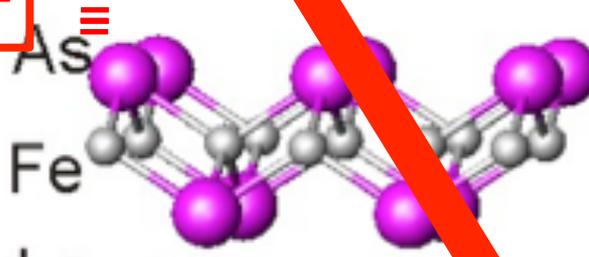
*how about  
very high  
pressure?*

2) Planes of anions only  
(MgB<sub>2</sub>)



*how about S=?  
r=1.84Å, vs  
r=1.40Å for O=*

3) Cation-anion tetrahedra  
(FeAs, FeSe, ...)



Cations should be small

*how about hydrogen?*

# Conventional superconductivity at 203 kelvin at high pressures in the sulfur hydride system

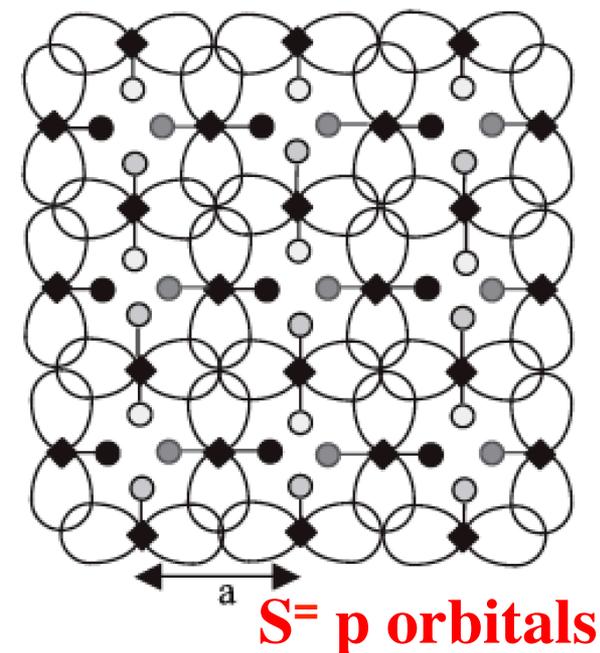
Nature, 2015

A. P. Drozdov<sup>1\*</sup>, M. I. Erements<sup>1\*</sup>, I. A. Troyan<sup>1</sup>, V. Ksenofontov<sup>2</sup> & S. I. Shylin<sup>2</sup>

Hole superconductivity in H<sub>2</sub>S and other sulfides under high pressure

Physica C 511 (2015) 45–49

J.E. Hirsch<sup>a,\*</sup>, F. Marsiglio<sup>b</sup>

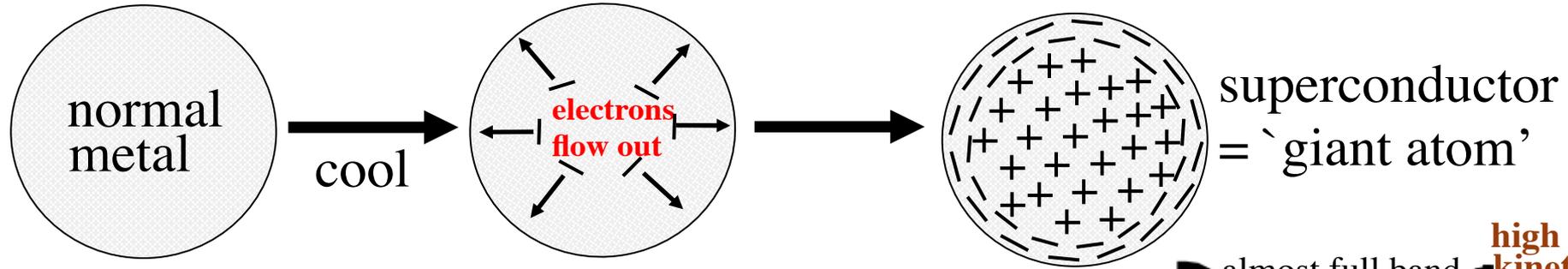


**holes conducting through  
p-orbitals of tightly packed  
S<sup>=</sup> anions in a planar structure**

Fig. 1. Left panel: proposed lattice structure for superconducting H<sub>2</sub>S. Planar *p* orbitals of the S<sup>=</sup> anions (rhombi) overlap, allowing for conduction of holes in the plane. The hydrogens bonded to the sulfurs are shown as circles with different shadings of grey indicating their distance to the plane of the paper. The two darker shadings indicate positions in front of the paper, the two lighter ones positions behind the paper. The two molecular bonds to a given S are at 90° to each other and to the corresponding *p* orbital in the plane, and the angle between the direction of the bond to the darker circle and the plane of the paper is denoted by  $\alpha$ , as indicated in the right panel of the figure.

# 1 mechanism for all materials: 'Hole superconductivity'

Superconductors **want to get rid of electrons** ==> expel electrons from interior to the surface



\* Which metals? those that have 'too many' electrons:

\* Which give highest  $T_c$ ? those that have most electrons:

*planes with negatively charged anions*

\* What does this explain? Meissner effect!

*Expulsion of magnetic field = expulsion of electrons*

\* What drives pairing, superconductivity and charge expulsion?

*Kinetic energy lowering*

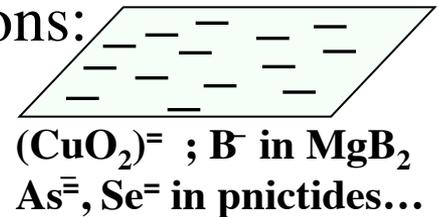
Microscopics: 'Dynamic Hubbard models'

Macroscopics: new London-like equations

Electric field in interior

Charge current near surface ( $B \neq 0$ )

Spin current near surface ( $B = 0$ )



# Summary:

- ❖ When a metal goes superconducting, electronic kinetic energy is lowered, e-e repulsion energy is lowered, e-ion energy increases
- ❖ Superconductors expel negative charge from their interior to a surface layer of width  $\lambda_L$
- ❖ A macroscopic electric field exists inside superconductors at  $T=0$
- ❖ A spin current flows near the surface of superconductors in the absence of applied external fields
- ❖ New London-like equations describe electrodynamics of charge and spin currents
- ❖ Explains dynamics of Meissner effect; BCS doesn't
- ❖ *This physics is common to all superconductors*
- ❖ *Its predictions can be tested experimentally*
- ❖ *Guidelines in the search for new high  $T_c$  superconductors: look for hole conduction through direct hopping between closely spaced negatively charged anions*